Analysis and prediction of the impact of COVID-19 on the global economy

using

FLOSS - R

submitted by

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15 October 2021

Contents

1	Abstract Introduction		2
2			3
3	Analysis and Results		4
	3.1	Data Collection	4
	3.2	Data Exploration	4
	3.3	Data Reformatting and Cleaning	5
	3.4	Data Preprocessing	6
	3.5	Data Analysis	8
		3.5.1 Linear Regression	8
		3.5.2 Artificial Neural Network (ANN)	
4	Cor	nclusion	19

Abstract

COVID-19 has had a profound impact on the lives of each individual. However, for countries as a whole, the economy has taken the hardest hit. This case study aims to analyze the Gross Domestic Product (GDP) and employment data of countries worldwide during the COVID-19 pandemic using statistical tools to predict their GDP loss. Predictions were made using factors like sector-wise GDP loss and sector-wise employment loss by applying multiple linear regression. Each data variable was tested for its significance to the regression model performance, and those with insignificant contributions were removed. Later, the regression results from the remaining variables were analyzed and explained. Then, a neural network was trained over the selected variables' data to compare its prediction results against the linear model. The comparison gave an insight into whether the data was suitable for a linear model or a non-linear model. Finally, the best among the two was chosen based on the prediction error percentage.

Introduction

Gross Domestic Product or GDP, is a monetary measure of the market value of all the final goods and services produced in a specific period [1]. GDP is often used as a metric for international comparisons and a broad measure of economic progress. It is widely considered one of the most powerful statistical indicators of national development [2]. Therefore, it is natural to study the impact of COVID-19 on a country's GDP to get an overall picture of the damage caused by it. In this case study, multiple linear regression and artificial neural network were used to predict the GDP loss of a country caused due to the pandemic. Each of the proposed statistical methods had its advantage; for example, the white-box linear regression model was beneficial in determining insignificant variables, whereas the neural network had a relatively more minor prediction error.

Analysis and Results

3.1 Data Collection

The COVID-19 Economic Impact Assessment data was collected for this case study from an online repository known as the ADB Data Library. The ADB Data Library is a platform that hosts publicly available data from the Asian Development Bank. The data obtained contains a measure of the potential economy and sector specific impact of the COVID-19 outbreak [3].

3.2 Data Exploration

The original dataset had the dimension 1566×10 and contained the following columns:

- **Economy**: Contains the country name.
- ADB Country Code: Contains the country code as assigned by ADB.
- **Sector**: Contains the economic sector from where the data was collected.
- Country 2018 GDP: Contains a country's GDP for the year 2018.
- Scenario: Contains the scenario based on which the GDP drop is predicted.
- as % of total GDP: Contains the GDP loss as a percentage of the total GDP.
- in \$ Mn: Contains the total income in denominations of \$1 million.
- Employment (in 000): Contains total number of people employed in counts of 1000s.
- as % of sector GDP: Contains percentage of sector GDP loss.
- as % of sector employment: Contains percentage of sector employment loss.

For further examination, the R programming language was used. R is a language and environment for statistical computing and graphics [4]. The dataset was imported into the R environment using the $read_xlsx()$ function of the readxl [5] package. Then a summary of the dataset was generated using the skim() function of the skimr [6] package as shown in Figure 3.1.

Figure 3.1: Summary of the original dataset.

From the summary, it was observed that the dataset contained nine character-type data columns and only one numeric-type data column which was incorrect as the columns "as % of total GDP", "in \$ Mn", "Employment (in 000)", "as % of sector GDP", and "as % of sector employment" must be of numeric type. Therefore, it was necessary to convert the data columns to appropriate format before performing any statistical analysis.

3.3 Data Reformatting and Cleaning

Since numeric values were incorrectly represented as characters, the following chunk of code was executed to reformat the data:

```
dforig_large$'in $ Mn'=sapply(dforig_large$'in $ Mn', replaceCommas<-function(x){
    x<-as.numeric(gsub("\\,", "", x))
}
dforig_large[,c(6:10)]=sapply(dforig_large[,c(6:10)],as.numeric)
dforig_large$'Country 2018 GDP'=sapply(dforig_large$'Country 2018 GDP',as.numeric)</pre>
```

After making changes, the column names were replaced with terms that could easily explain the data associated with the respective column.

After changing the column names, the dataset's summary was again generated and analyzed as shown in Figure 3.2.

```
values
dforig_large
1576
10
Column type frequency:
character
numeric
Group variables
                                  None
   Variable type: character --
A tibble: 4 x 8
# A tibble: 4 x 8 skim_variable
                      * <chr>
1 Country Name
2 ADB Country Code
3 Sector
4 Scenario
  O
- Variable type: numeric ------
A tibble: 6 x 11
skim_Variable n_missing
                                                              n_missing complete_rate
<int> <dbl>
                                                                                                                               1 2018 GDP
                                                       1
0.668
1 2018 GDP
2 Total GDP Loss
3 In Million
4 Employment in 1000s
5 Sector GDP Loss
6 Sector Employment Loss
                                                        0.668
0.640
```

Figure 3.2: Summary of the reformatted dataset.

It can be observed from the summary that the dataset contained missing values. Those missing values were later removed using the na.omit() function as shown below:

```
1 dforig = na.omit(dforig_large)
```

The cleaned dataset contained 1008 row entries. Now the dataset need to go through various preprocessing steps to make it suitable for analysis.

3.4 Data Preprocessing

To make the data suitable for analysis, the following steps were performed:

• Rephrasing the *Scenario* column string values for easier understanding.

• Rephrasing the Sector column string values for easier understanding.

• Removing row entries containing the value "_ALL" in the Sector column.

```
1 dfnn=dfnn[-c(which(dfnn$Sector=="_All")),]
```

• Converting character variables to factors.

```
1 dforig$Sector=as.factor(dforig$Sector)
2 dforig$Scenario=as.factor(dforig$Scenario)
```

• Selecting relevant columns for data analysis.

```
dfnn=as_tibble(select(dforig, 'Percentage of Total GDP Loss', Sector, Scenario, 'Percentage of Sector GDP Loss', 'Percentage of Sector Employment Loss'))
```

• Changing the data type of character columns containing numbers to numeric.

```
dfnn$'Percentage of Total GDP Loss'=sapply(dfnn$'Percentage of Total GDP Loss', as.numeric)
```

• Removing row entries with non-negative value for percentage change in GDP to perform an independent analysis.

```
1 df_pos_gdp=dfnn[which(dfnn$'Total GDP Loss'>=0),]
2 dfnn=dfnn[-c(which(dfnn$'Total GDP Loss'>=0)),]
```

• Normalizing the dependent variable, "Total GDP Loss", using logarithmic transformation.

```
1 nrmls=function(x)
2 {
3    return (log(abs(x)))
4 }
5 dfnn$'Total GDP Loss'=sapply(dfnn$'Total GDP Loss', nrmls)
```

• Logarithmic transformation may output some values as "Inf" or infinity. Hence, it is necessary to remove row entries containing infinite value (if any).

```
1 real_index=which(is.infinite(dfnn$'Percentage of Total GDP Loss') == FALSE)
2 dfnn=dfnn[c(real_index),]
```

• Splitting data in a ratio of 3:1 for training and testing respectively, to apply the linear regression model over it.

```
## 75% for training data
smp_size <- floor(0.75* nrow(dfnn))
## Set random seed for reproducibility
set.seed(123)
## Creating a list of random training indices
train_ind <- sample(seq_len(nrow(dfnn)), size = smp_size)
## Segregating training and testing data
train <- dfnn[train_ind, ]
test <- dfnn[-train_ind, ]</pre>
```

The data preprocessing resulted in a normally distributed dependent variable as shown in Figure 3.3:

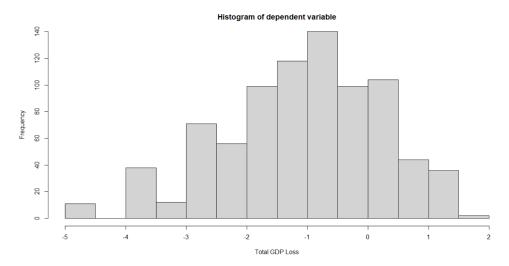


Figure 3.3: Histogram of the dependent variable.

3.5 Data Analysis

3.5.1 Linear Regression

Linear regression is a linear approach for modeling the relationship between a scalar response and one or more explanatory variables (also known as dependent and independent variables respectively). The case of a single explanatory variable is called simple linear regression; for more than one, the process is called multiple linear regression [7]. For the case study, multiple linear regression was utilized.

Original Model

The multiple linear regression model was trained to predict GDP loss, given the sector, scenario and sector-wise GDP loss. The command used to train the model is as follows:

```
1 lm_best=lm('Percentage of Total GDP Loss'~'Percentage of Sector GDP Loss'*Sector+ Scenario,data=train)
```

The following is the summary of the obtained linear model:

```
Call:
lm(formula = 'Total GDP Loss' ~ 'Sector GDP Loss' * Sector +
    Scenario, data = train)
Residuals:
            10 Median
                             3Q
                                     Max
-3.6064 -0.3783 0.0942 0.4550
                                 2.2762
Coefficients:
                                                    Estimate Std. Error t value Pr(>|t|)
(Intercept)
                                                    -2.182295
                                                              0.152073 -14.350 < 2e-16
                                                                0.036889 -4.475 9.11e-06 ***
                                                   -0.165087
 Sector GDP Loss
SectorBusiness and Trade
                                                    1.205159
                                                                0.173470
                                                                           6.947 9.59e-12 ***
SectorHotels and Restaurant
                                                    -0.030008
                                                                0.157658
                                                                          -0.190 0.849108
                                                                0.173703
SectorLight/Heavy Manufacturing
                                                    0.669449
                                                                           3.854 0.000129 ***
SectorTransport services
                                                   -0.047055
                                                                0.155710
                                                                          -0.302 0.762607
ScenarioAdditional impact (Short term)
                                                   -0.342538
                                                                0.124331
                                                                          -2.755 0.006044 **
ScenarioLong term effects
                                                   -0.298746
                                                                0.102151
                                                                          -2.925 0.003578 **
                                                   -0.244673
                                                                0.101781
ScenarioShort term effects
                                                                          -2.404 0.016519
Sector GDP Loss`:SectorBusiness and Trade
Sector GDP Loss`:SectorHotels and Restaurant
                                                                          -5.300 1.62e-07 ***
                                                   -0.260295
                                                                0.049110
                                                    0.003093
                                                                0.038027
                                                                           0.081 0.935200
Sector GDP Loss: SectorLight/Heavy Manufacturing -0.231902
                                                                0.051978
                                                                          -4.462 9.69e-06 ***
Sector GDP Loss`:SectorTransport services
                                                    0.035370
                                                               0.037311
                                                                           0.948 0.343508
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 0.762 on 609 degrees of freedom
Multiple R-squared: 0.6711,
                                Adjusted R-squared:
F-statistic: 103.5 on 12 and 609 DF, p-value: < 2.2e-16
```

Figure 3.4: Summary of the obtained multiple linear regression model.

The results of the linear regression model were good but the contribution of some variables was insignificant which can be explained as follows:

- Hotels and Restaurant (Sector) Restrictions on hotel and restaurant services were removed sooner than others, so that people could get food delivered to them as it could reduce the rate of the spread of infection. Hence, the associated data variable was quite useless in predicting the GDP drop.
- Transport services (Sector) During the pandemic, there were restrictions imposed on the travel of citizens, within and outside a country. However, compared to other sectors, transport sector got disrupted only for a short time. Hence, it did not contribute much to the prediction of GDP loss.

The q-q (quantile-quantile) plot is a probability plot, which is a graphical method for comparing two probability distributions by plotting their quantiles against each other [8]. The q-q plot of the obtained linear model's residuals is given below:

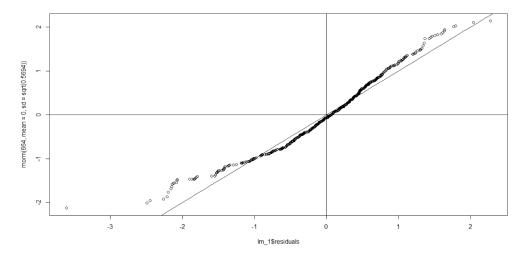


Figure 3.5: q-q plot of residuals.

It can be observed that the residuals approximately follow a normal distribution. Another method to check for normality is performing KS test over the residuals. The KS test is a non-parametric test that is used to decide if a sample comes from a population having a particular distribution [9]. Since the KS test is more precise as compared to the graphical method for testing normality, it was used to verify the previous findings. The test was performed using the following command:

```
1 LcKS(lm_best$residuals,"pnorm")
```

The resulting "p-value" was 2e-04 which confirmed that the residuals were gaussian.

The accuracy of the obtained model was determined by performing the following operations:

- Computing the Root Mean Square Error (RMSE) between predicted values and original test dataset values.
- Computing the minimum and maximum deviation of predicted values from the original values.

All metrics along with the code are shown in Figure 3.6:

```
"``{r warning=FALSE}
RMSE(predlm_best,test$`Total GDP Loss`)

[1] 1.895851

"``{r warning=FALSE}
min(abs(predlm_best-test$`Total GDP Loss`))

[1] 0.004966997

"``{r warning=FALSE}
max(abs(predlm_best-test$`Total GDP Loss`))

[1] 9.198139
```

Figure 3.6: Model's accuracy measurement results.

The predictions obtained from the model were visualized by plotting the predicted values (red) over the test dataset values. The visualization indicated that the obtained linear regression model did not accurately fit the given data:

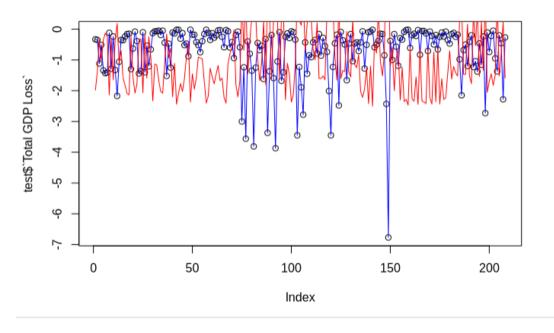


Figure 3.7: Predicted values versus original values.

Improved Model 1

The previous attempt showed that certain variables tend to affect the accuracy of the linear model. Therefore, another attempt was made after removing the row entries associated with "Hotels and Restaurant" and "Transport services" data values of the "Sector" column as their contribution to the previously obtained model was

insignificant. The command for generating a linear regression model remained the same. The following is the summary of the newly obtained model:

```
Call:
lm(formula = 'Total GDP Loss' ~ 'Sector GDP Loss' * Sector +
    Scenario, data = train)
Residuals:
             10 Median
    Min
                             30
                                    Max
-2.4627 -0.3389
                0.0037 0.4263
                                 2.1806
Coefficients:
                                                    Estimate Std. Error t value Pr(>|t|)
(Intercept)
                                                   -2.029224
                                                               0.167534 -12.112
                                                                                 < 2e-16
                                                                         -4.861 1.74e-06 ***
 Sector GDP Loss
                                                   -0.177075
                                                               0.036427
                                                               0.159716
                                                                          6.509 2.51e-10
SectorBusiness and Trade
                                                    1.039668
SectorLight/Heavy Manufacturing
                                                    0.570549
                                                               0.163882
                                                                          3.481 0.000559
ScenarioAdditional impact (Short term)
                                                    0.004559
                                                               0.157959
                                                                          0.029 0.976990
ScenarioLong term effects
                                                   -0.390089
                                                               0.119868
                                                                         -3.254 0.001243
ScenarioShort term effects
                                                   -0.400104
                                                               0.130090
                                                                         -3.076 0.002260 **
 Sector GDP Loss`:SectorBusiness and Trade
                                                                         -6.036 3.90e-09 ***
                                                   -0.271501
                                                               0.044979
Sector GDP Loss: SectorLight/Heavy Manufacturing -0.225487
                                                               0.048148
                                                                         -4.683 4.00e-06
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 0.6996 on 363 degrees of freedom
Multiple R-squared: 0.6988,
                                Adjusted R-squared: 0.6921
F-statistic: 105.3 on 8 and 363 DF, p-value: < 2.2e-16
```

Figure 3.8: Summary of the improved linear regression model.

From the above model summary, it can be observed that "Scenario Additional impact (Short term)" is an insignificant variable. It is understandable as short term impact may not affect GDP much. The q-q plot of the model's residuals is shown below:

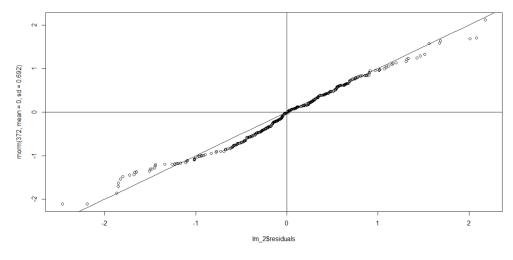


Figure 3.9: q-q plot of residuals.

All metrics associated with the improved model's accuracy along with the code are shown in Figure 3.10:

```
"``{r warning=FALSE}
RMSE(predlm_sgnf,test$`Total GDP Loss`)
"``
[1] 0.7697049

'```{r warning=FALSE}
min(abs(predlm_sgnf-test$`Total GDP Loss`))
"``
[1] 0.0002347793

'```{r warning=FALSE}
max(abs(predlm_sgnf-test$`Total GDP Loss`))
"``
[1] 2.266151
```

Figure 3.10: Improved model's accuracy measurement results.

The predictions of the improved model are shown below:

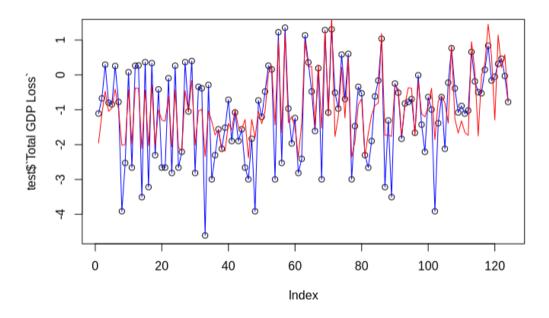


Figure 3.11: Improved model's predicted values versus original values.

Compared to the previous model, the improved model produced far better results. However, it was necessary to check if removing the last insignificant variable will produce better results or not.

Improved Model 2

The final linear regression model was trained after removing the only remaining insignificant variable from the dataset. Following is the summary of the final model:

```
Call:
lm(formula = `Total GDP Loss` ~ `Sector GDP Loss` * Sector +
    Scenario, data = train)
Residuals:
                1Q
                      Median
-2.34449 -0.37810 0.02369
                               0.45598
                                         2.26262
Coefficients:
                                                       Estimate Std. Error t value Pr(>|t|)
(Intercept)
                                                        -2.15116
                                                                     0.16939 -12.699
                                                                                        < 2e-16
 Sector GDP Loss
                                                                                      6.55e-09 ***
                                                        -0.22387
                                                                     0.03753
                                                                               -5.965
SectorBusiness and Trade
                                                         1.28251
                                                                                7.234
SectorLight/Heavy Manufacturing
                                                        0.78689
                                                                     0.18648
                                                                                4.220 3.20e-05
ScenarioLong term effects
                                                        -0.48264
                                                                     0.11780
                                                                               -4.097
                                                                                      5.32e-05
ScenarioShort term effects
`Sector GDP Loss`:SectorBusiness and Trade
                                                                               -3.680 0.000274 ***
                                                        -0.47219
                                                                     0.12831
                                                                     0.04786
                                                        -0.21389
                                                                               -4.469 1.09e-05
 Sector GDP Loss`:SectorLight/Heavy Manufacturing -0.17390
                                                                               -3.372 0.000838
                                                                     0.05157
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.718 on 316 degrees of freedom
Multiple R-squared: 0.7134, Adjusted R-squared: 0.7071
F-statistic: 112.4 on 7 and 316 DF, p-value: < 2.2e-16
```

Figure 3.12: Summary of the final linear regression model.

The q-q plot of the final model's residuals is shown below:

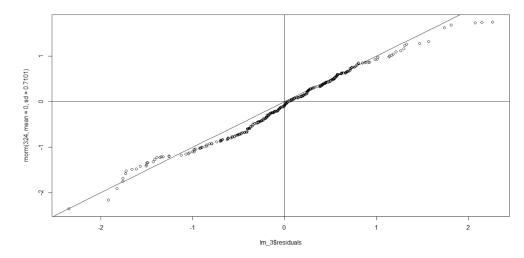


Figure 3.13: q-q plot of the final model's residuals.

The residuals follow a normal distribution indicating improvement in comparison to previous attempts. Now, it is necessary to check the residuals for homoscedasticity. In statistics, a sequence of random variables is homoscedastic if all its random variables have the same finite variance. Uneven variances may lead to biased and skewed results [10]. To check for homoscedasticity, the plot of the square of residuals was examined as shown below:

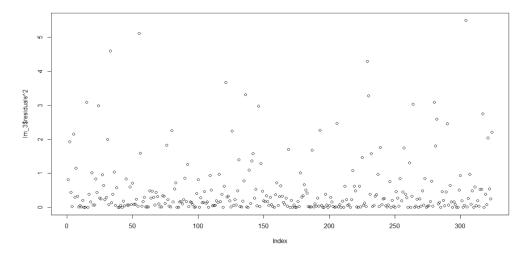


Figure 3.14: Squared residual plot of the final model.

The above plot was shown to mentor for examination and he concluded that the plot indicates heteroscedasticity. To remove it, an Artificial Neural Network (ANN) model was later implemented over the dataset associated with the final model. The final linear regression model's accuracy is shown below:

```
"" {r warning=FALSE}
RMSE(predlm_sgnf_2,test$`Total GDP Loss`)

[1] 0.7906823
"" {r warning=FALSE}
min(abs(predlm_sgnf_2-test$`Total GDP Loss`))

[1] 0.002623887
"" {r warning=FALSE}
max(abs(predlm_sgnf_2-test$`Total GDP Loss`))

[1] 2.162498
```

Figure 3.15: Final model's accuracy measurement results.

While there was no improvement in the RMSE of the model, the maximum difference between the predicted and the actual values got reduced, thus improving fit. The predictions of the final model are shown below:

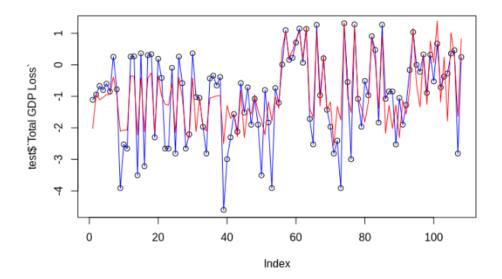


Figure 3.16: Final model's predicted values versus original values.

The final model is the best among all. However, RMSE can be further improved by making use of an ANN model as described in the following section.

3.5.2 Artificial Neural Network (ANN)

An artificial neural network takes an input vector of p variables $X = (X1, X2, ..., X_p)$ and builds a non-linear function f(X) to predict the response Y [11]. The nnet [12] package of R was used to create the ANN model. The number of hidden units were set to six after numerous experiments. To ensure reproducibility of the results, the command to train an ANN was made to run 1000 times with the iteration number set as the random seed. Finally, the best ANN model with RMSE lower than that of the final linear regression model was selected and saved in the form of an .rds file for later use. The code to train and save an ANN model is shown below:

```
ANN_results <- NULL
   for(i in 1:1000)
3
 4
     set.seed(i)
 5
     nnet_gdp=nnet('Total GDP Loss'~'Sector GDP Loss'+Sector+Scenario,train,size=6,
         maxit=2000,linout=TRUE, abstol=0.000001,trace=FALSE)
6
     pred_nnet=predict(nnet_gdp,test)
 7
     new_rm=RMSE(pred_nnet,test$'Total GDP Loss')
8
     ANN_results <- rbind(ANN_results,c(i,new_rm))
9
10 ANN_selected=ANN_results[which(as.data.frame(ANN_results)$V2 == min(as.data.frame(
       ANN_results)$V2)),]
11
   ANN selected
12
   set.seed(ANN_selected[1])
13 nnet_gdp=nnet('Total GDP Loss'~'Sector GDP Loss'+Sector+Scenario,train,size=6,maxit
       =2000,linout=TRUE, abstol=0.000001,trace=FALSE)
  pred_nnet=predict(nnet_gdp,test)
15 new_rm=RMSE(pred_nnet,test$'Total GDP Loss')
16 saveRDS(nnet_gdp, "Best_NNET_GDP.rds")
   best_rmse=new_rm
```

The best ANN model had the random seed set to 819 and its accuracy measurements are as follows:

```
best_rmse

[1] 0.5474239

'``{r}
min(abs(pred_nnet-test$`Total GDP Loss`))

[1] 0.002074419

'``{r}
max(abs(pred_nnet-test$`Total GDP Loss`))

[1] 1.991076
```

Figure 3.17: ANN model's accuracy measurement results.

The ANN model gave better results as compared to the final linear regression model. The squared residual plot associated with the ANN model is shown below:

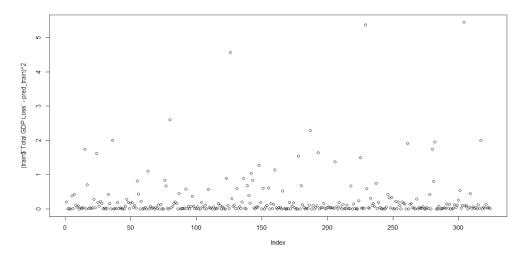


Figure 3.18: Squared residuals plot associated with the neural network model.

The above plot was reviewed by mentor and he concluded that the plot seems to indicate approximate homoscedasticity. The model fits the test data very well, as shown by the following figure:

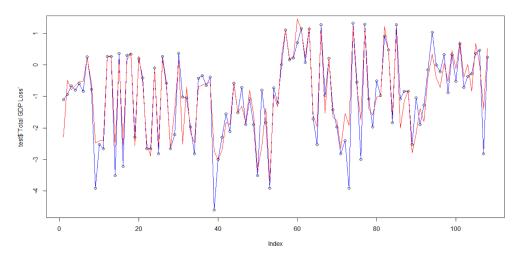


Figure 3.19: ANN model's predicted values versus original values.

Conclusion

The case study attempted to explore the impact of COVID-19 on the GDP of various countries. The obtained statistical models have accurately predicted the GDP loss. Besides estimating the underlying pattern of the given data, the linear regression models also helped discover insignificant variables in the dataset. Later, an artificial neural network model was also trained for better accuracy and fit. As the final linear regression model predicted the actual values with a minor error. It seems like a good choice for predicting the GDP loss as it has the advantage of being a white-box approach. On the other hand, a neural network may provide better results but it is a black-box approach.

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