Two dimensional Probability Distribution

Introduction

Given two random variables that are defined on the same probability space, the joint probability distribution is the corresponding probability distribution on all possible pairs of outputs. The joint distribution can just as well be considered for any given number of random variables.

Definition:

Let S be the sample space. Let X = X(S) & Y = Y(S) be two functions each assigning a real number to each outcome $s \in S$. Then (X, Y) is a two dimensional random variable.

Types of random variables

- Discrete R.V.'s
- Continuous R.V.'s

Discrete R.V.'s (Two Dimensional Discrete R.V.'s)

If the possible values of (X, Y) are finite, then (X, Y) is called a two dimensional discrete R.V. and it can be represented by (x_i, y_j), i = 1,2,….,m and j=1,2,…n

Joint Probability Function of discrete R.V.'s X and Y

The function $P(X = x_i, Y = y_j) = P(x_i, y_j)$ is called the joint probability function for discrete random variable X and Y is denoted by p_{ij}.

Note:

1. $P(X = x_i, Y = y_j) = P[(X = x_i)\cap(Y = y_j)] = p_{ij}$
2. It should satisfies the following conditions

   (i) $p_{ij} \geq 0$, for all i, j (ii) $\Sigma_j\Sigma_i\, p_{ij} = 1$

Marginal Probability Function

- If the joint probability distribution of two random variables X and Y is given then the marginal probability function of X is given by

  $\Sigma_j\, P(X = x, Y = y_j)$

- If the joint probability distribution of two random variables X and Y is given then the marginal probability function of Y is given by

  $\Sigma_i\, P(X = x_i, Y = y)$

Procedure:

- Input the data set
- Determine the probabilities of the random variable using R functions
- Visualize the joint probability distribution using R functions

Note: Please make sure that the following package is already installed
"scatterplot3d"

Problem:

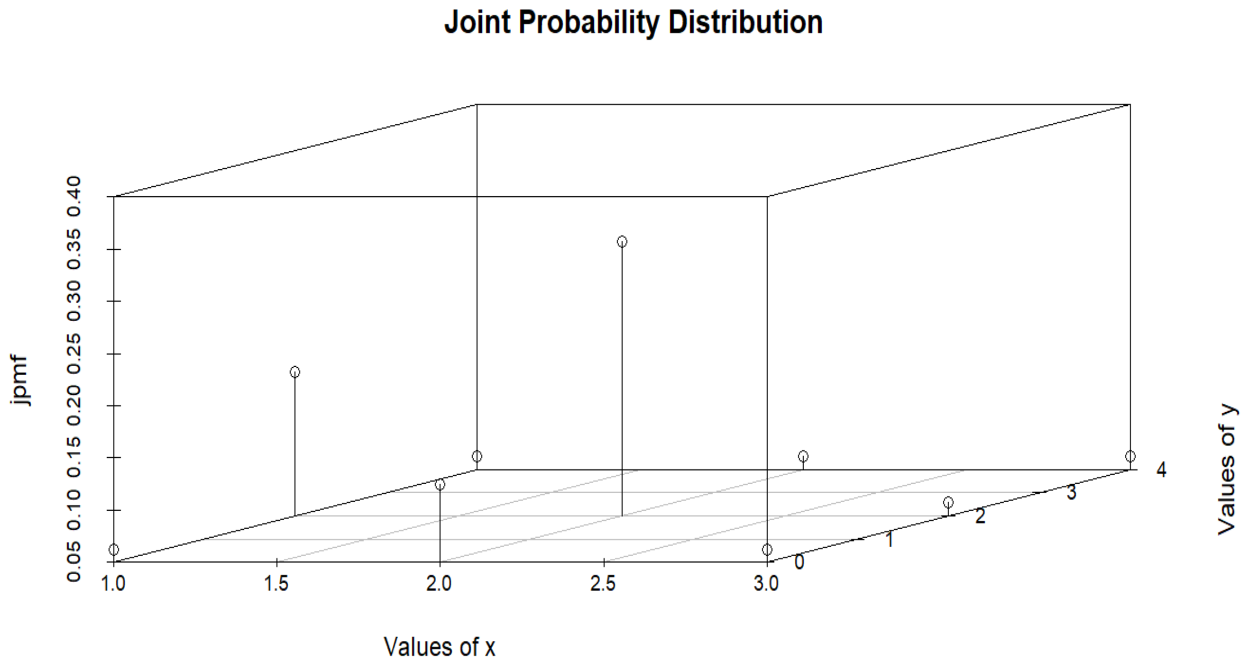
Consider the following joint distribution of X and Y distributions.

Y \ X	1	2	3
0	1/16	3/16	1/16
2	2/16	5/16	1/16
4	1/16	1/16	1/16

Find (i) Marginal distributions of X and Y
(ii) Visualize the joint probability distribution (X,Y)

Code and Results:

```
# input the data
xy=c(1/16,3/16,1/16,2/16,5/16,1/16,1/16,1/16,1/16)
xy

## [1] 0.0625 0.1875 0.0625 0.1250 0.3125 0.0625 0.0625 0.0625 0.0625

# arrange in matrix form
jpmf=matrix(xy,nrow=3,byrow=TRUE)
jpmf

##        [,1]   [,2]   [,3]
## [1,] 0.0625 0.1875 0.0625
## [2,] 0.1250 0.3125 0.0625
## [3,] 0.0625 0.0625 0.0625

# Marginal probability distribution of x
colSums(jpmf)

## [1] 0.2500 0.5625 0.1875

# Marginal probability distribution of y
rowSums(jpmf)

## [1] 0.3125 0.5000 0.1875

# label the values of x
colnames(jpmf) <- c("1","2","3")
# label the values of y
rownames(jpmf) <- c("0","2","4")
# visualize the joint probability distribution
c1=c(1,2,3,1,2,3,1,2,3)
c2=c(0,0,0,2,2,2,4,4,4)
library(scatterplot3d)
scatterplot3d(c1,c2,jpmf,type='h',xlab='Values of x', ylab='Values of
y',main="Joint Probability Distribution")
```


Joint Probability Distribution
jpmf
0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40
1.0 1.5 2.0 2.5 3.0
Values of x
0 1 2 3 4
Values of y