Discrete Probability Distribution

Introduction:
A discrete distribution is one in which the data can only take on certain values, for example integers. For a discrete distribution, probabilities can be assigned to the values in the distribution. These distributions model the probabilities of random variables that can have discrete values as outcomes. Example: Binomial distribution, Poisson distribution

A binomial distribution is a discrete probability distribution that gives the success probability in n Bernoulli trials. The probability of getting a success is given by p. It is represented as X ~ Binomial(n, p). The pmf is given as follows:

$$P(X = x) = nC_{\,x} p^x (1-p)^{n-x}, \qquad x = 0,1,2,\ldots,n$$

Poisson distribution is a discrete probability distribution that is widely used in the field of finance. It gives the probability that a given number of events will take place within a fixed time period. The notation is written as X ~ Pois(λ), where λ>0. The pmf is given by the following formula:

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad x = 0,1,2,\ldots$$

Procedure:

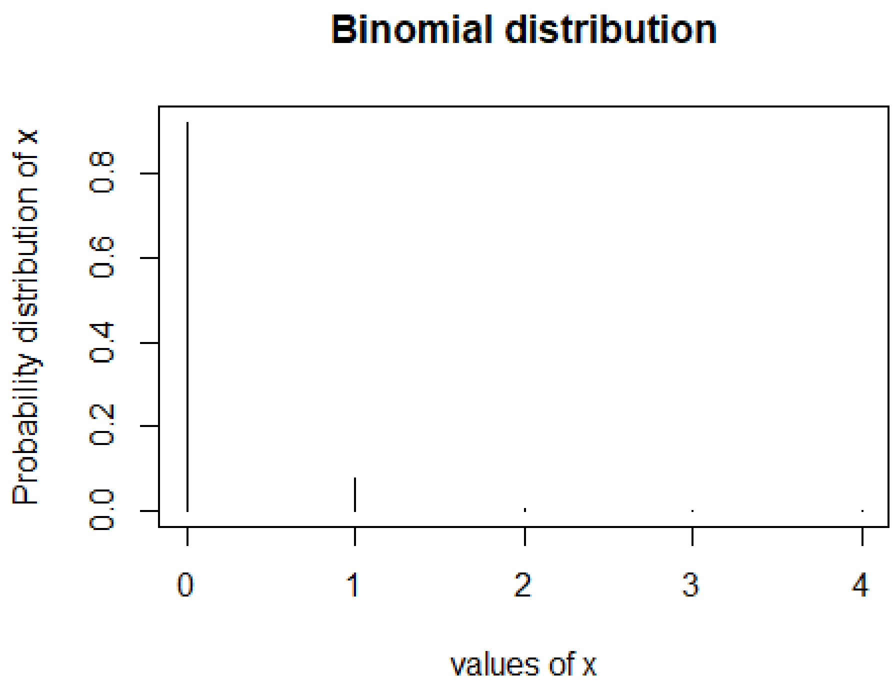
- Import the data set
- Determine the probabilities of the random variable using Binomial distribution in R
- Determine the probabilities of the random variable using Poisson distribution in R
- Visualize the probability distribution using R functions

Problem:
Four coins are tossed simultaneously. What is the probability of getting (i) 2 heads (ii) atleast 2 heads (iii) atmost 2 heads (iv) Expectation of x (v) Variance of x (vi) Visualize the probability distribution

Code and Results:

```r
# number of trials
n=4
n

## [1] 4

#probability of success
p=0.02
p

## [1] 0.02

# (i) probability of getting exactly 2 heads
dbinom(2,n,p)
```

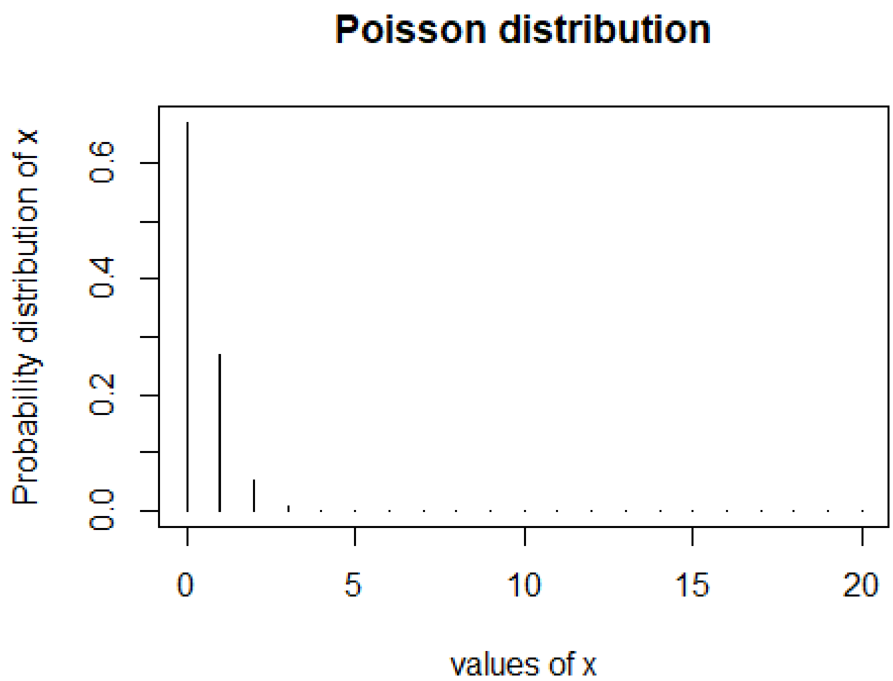
```
## [1] 0.00230496

# (ii) probability of getting atleast 2 heads
sum(dbinom(2:4,n,p))

## [1] 0.00233648

#or
1-pbinom(1,n,p)

## [1] 0.00233648

# (iii) probability of getting atmost 2 heads
sum(dbinom(0:2,n,p))

## [1] 0.9999685

# or
pbinom(2,n,p)

## [1] 0.9999685

#(iv) Expectation of x
x=0:n
px=dbinom(x,n,p)
Ex=weighted.mean(x,px)
Ex

## [1] 0.08

# (v) variance of x
Varx=weighted.mean(x*x,px)-(weighted.mean(x ,px))^2
Varx

## [1] 0.0784

# (vi) Visualize probability distribution
plot(x,px,type="h",xlab="values of x",ylab="Probability distribution of
x",main="Binomial distribution")
```



Problem:
A manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 20 and find the number of boxes containing (i) at least 2 defective (ii) exactly 2 defective (iii) at most 2 defective pins in a consignment of 1000 boxes (iv) plot the distribution (v) E(x) (vi) Variance of X?

```
#Poisson Distribution
# number of trails
m=20
m

## [1] 20

# probability of success
ps=0.02
# poisson parameter
lambda=m*ps
lambda

## [1] 0.4

#at least 2 defectives
p1=sum(dpois(2:m,lambda))
p1
```

```
## [1] 0.06155194

# (i) number of boxes containing at least 2 defectives
round(1000*p1)

## [1] 62

#exactly 2 defectives
p2=dpois(2,lambda)
p2

## [1] 0.0536256

# (ii) number of boxes containing exactly 2 defectives
round(1000*p2)

## [1] 54

#at most 2 defectives
p3=sum(dpois(0:2, lambda))
p3

## [1] 0.9920737

# (iii) number of boxes containing at most 2 defectives
round(1000*p3)

## [1] 992

# (iv) plot the distribution
x1=0:m
px1=dpois(x1,lambda)
plot(x1,px1,type="h",xlab="values of x",ylab="Probability distribution of
x",main="Poisson distribution")
```




```
#(v) E(x)
Ex1=weighted.mean(x1,px1)
Ex1

## [1] 0.4

# (vi) variance of x
Varx1=weighted.mean(x1*x1,px1)-(weighted.mean(x1 ,px1))^2
Varx1

## [1] 0.4
```