Experiment no 4 Title of Experiment: Poisson distribution

Abstract

A Poisson process is where discrete events occur in a continuous, but finite interval of time or space in R. The Poisson distribution evaluates cumulative probability of less than or equal to 'q' successes. The analysis here can be done by three ways :

– Numerical Analysis
– R- Programming Analysis
– Graphical Analysis

Introduction: Poisson distribution has the following characteristics

– Infinite number of trials and outcomes
– The mean of the distribution is the same for all intervals.
– No. of occurrence in any given interval independent of other and has the predictive power of occurrences per unit, time, space.

The Poisson distribution may be useful to model events as follows:

– The number of patients arriving in an emergency room between 10 and 11 pm
– The number of photons hitting a detector in a particular time interval.

Numerical Analysis

Poisson distribution states that the probability of observing k events in an interval is given by the equation.

$$\boxed{P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!}}$$

where,

- Lambda is the average number of events per interval
- e is the number 2.71828... (Euler's number) the base of the natural logarithms.
- k takes values 0, 1, 2, ...
- k! = k × (k − 1) × (k − 2) × ... × 2 × 1 is the factorial of k.

This equation is the probability mass function (PMF) for a Poisson distribution. Also for average number of events lambda at timerate r for the events to happen, then lambda= rt is considered to find the Poisson distribution.

$$\boxed{P(k \text{ events in interval } t) = \frac{(rt)^k e^{-rt}}{k!}}$$

Experiment: Poisson Evaluation for occurrence of river floods in 100 years

On a particular river, overflow floods occur once every 100 years on average. Calculate the probability of k = 0, 1, 2, 3, 4, 5, or 6 overflow floods in a 100-year interval, assuming the Poisson model is appropriate.

Because the average event rate is one overflow flood per 100 years,λ = 1

$$P(k \text{ overflow floods in 100 years}) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{1^k e^{-1}}{k!}$$
$$P(k = 0 \text{ overflow floods in 100 years}) = \frac{1^0 e^{-1}}{0!} = \frac{e^{-1}}{1} \approx 0.368$$
$$P(k = 1 \text{ overflow flood in 100 years}) = \frac{1^1 e^{-1}}{1!} = \frac{e^{-1}}{1} \approx 0.368$$
$$P(k = 2 \text{ overflow floods in 100 years}) = \frac{1^2 e^{-1}}{2!} = \frac{e^{-1}}{2} \approx 0.184$$

The table below gives the probability for 0 to 6 overflow floods in a 100-year period.

k	*P*(*k* overflow floods in 100 years)
0	0.368
1	0.368
2	0.184
3	0.061
4	0.015
5	0.003
6	0.0005

R – Programming Analysis

Essential terms: Density, distribution function, quantile function and random generation for the Poisson distribution with parameter λ.

Usage

dpois(x, λ, log = FALSE)

ppois(q, λ, lower.tail = TRUE, log.p = FALSE)

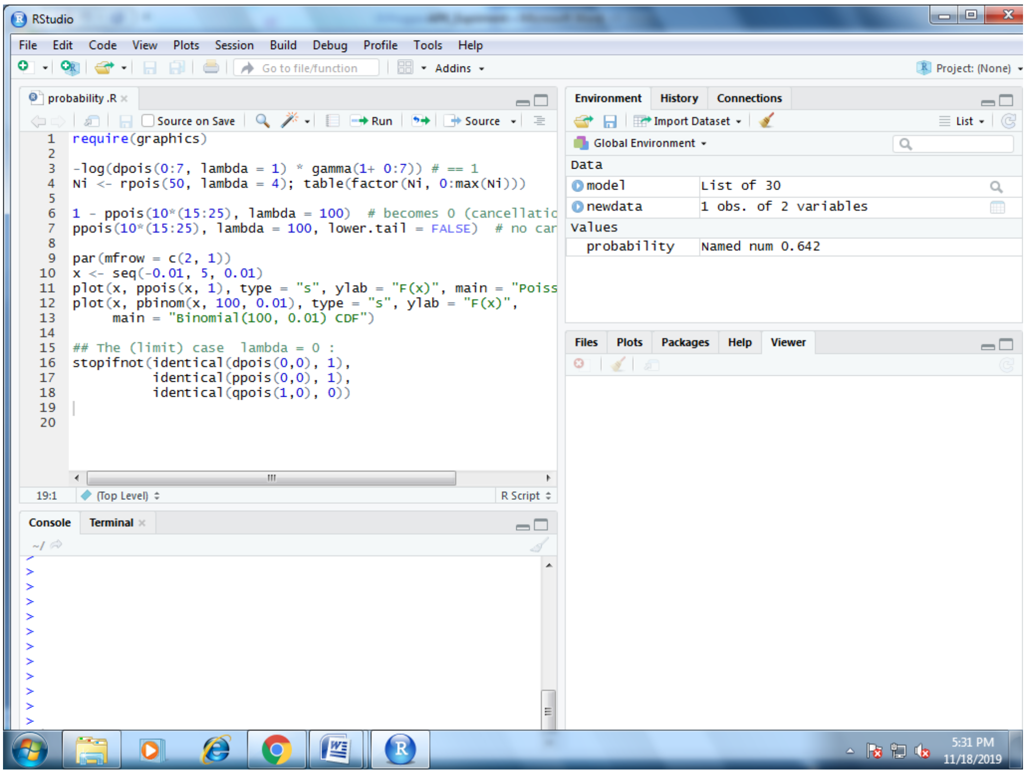
qpois(p, λ, lower.tail = TRUE, log.p = FALSE)

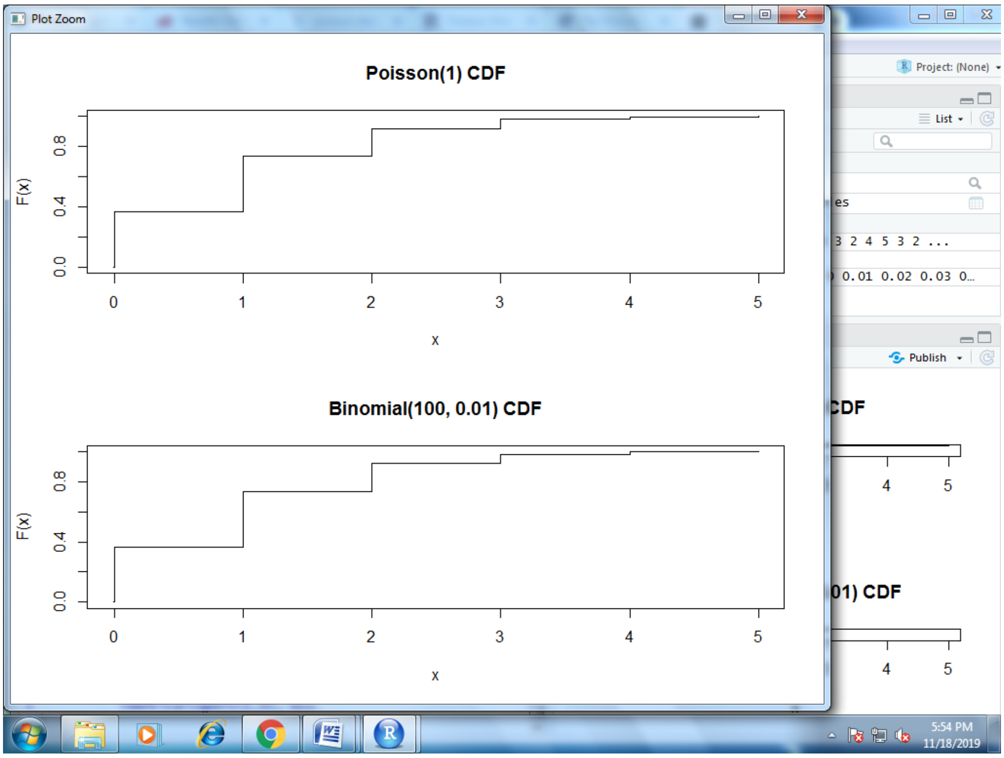
rpois(n, λ)

Arguments

X	vector of (non-negative integer) quantiles.
Q	vector of quantiles.
P	vector of probabilities.
N	number of random values to return.
Lambda (λ)	vector of (non-negative) means.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X ≤ x], otherwise, P[X > x].

Numerical and Graphical Analysis with R – Programming





Conclusion –On a particular river, overflow floods occur once every 100 years on average of probability 64.2%.