

R Textbook Companion for
Introduction to Mathematical Statistics
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Book Description

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R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

Contents

List of R Codes	4
1 Probability And Distributions	5
2 Multivariate Distributions	29
3 Some Special Distributions	35
4 Some Elementary Statistical Inferences	40
5 Consistency and Limiting Distributions	53
6 Maximum Likelihood Methods	56
8 Optimal Tests of Hypotheses	57
10 Nonparametric and Robust Statistics	60
11 Bayesian Statistics	70

List of R Codes

Exa 1.1.1	Coin Toss	5
Exa 1.1.2	Dice Sample Space	5
Exa 1.1.3	Dice Relative Frequency	5
Exa 1.2.3	Union	6
Exa 1.2.4	Union	6
Exa 1.2.5	Union	6
Exa 1.2.6	Union	7
Exa 1.2.8	Intersection of Sets	7
Exa 1.2.10	Intersection of Sets	7
Exa 1.2.12	Venn Diagram	8
Exa 1.2.13	Coin Sample Space	8
Exa 1.2.15	Set Complement	9
Exa 1.2.16	Set Operations	9
Exa 1.2.17	De Morgans Law	10
Exa 1.2.18	Set Functions	10
Exa 1.2.21	Functions	12
Exa 1.2.23	1D Set Integration	12
Exa 1.3.1	Set Probability	13
Exa 1.3.2	Set Probability	13
Exa 1.3.4	Poker Question	14
Exa 1.4.1	Conditional Probabilty	14
Exa 1.4.2	Conditional Probabilty Chip Problem	15
Exa 1.4.3	Conditional Probabilty Card Problem	15
Exa 1.4.4	Cards In Order	16
Exa 1.4.5	Bayes Theorem	16
Exa 1.4.6	Plant Problem	17
Exa 1.4.7	Bayes Theorem Child Abuse Problem	17
Exa 1.4.8	Sum of 2 Die	18

Exa 1.4.9 Mutual Independence	18
Exa 1.4.10 Coin Probability	19
Exa 1.4.11 Mutually Independent Events	19
Exa 1.5.1 Random Variable Pmf	20
Exa 1.5.2 Probability Density Function	20
Exa 1.5.3 Step Function Plot	21
Exa 1.5.4 Cumulative Distribution Function	21
Exa 1.5.5 Mechanical Part CDF Problem	21
Exa 1.5.6 Cumulative Distribution Function	22
Exa 1.5.7 Probability Mass Function	22
Exa 1.5.8 Probability Density Function	22
Exa 1.6.2 Fuse Problem	23
Exa 1.7.1 Circle Probability Problem	23
Exa 1.7.2 Telephone Probability Problem And Graph	23
Exa 1.7.6 Cumulative Distribution Function	24
Exa 1.7.7 Company Loss Problem	24
Exa 1.8.2 Random Variable	25
Exa 1.8.3 Probability Density Function	25
Exa 1.8.4 Expectation Problem	25
Exa 1.8.5 Expectation Problem	26
Exa 1.8.6 Expectation Problem	26
Exa 1.8.7 Expectation Chip Problem	27
Exa 1.9.1 Mean And Variance	27
Exa 1.10.1 Chebyshev Inequality	28
Exa 2.1.2 Multivariate Random Variable	29
Exa 2.1.3 Chips From Bowl Problem	29
Exa 2.1.4 Joint PDF Problem	30
Exa 2.1.5 Multivariate Expectation Problem	30
Exa 2.1.6 Multivariate Expectation Problem	31
Exa 2.3.1 Mean Variance And Probability	32
Exa 2.3.2 Mean And Variance	32
Exa 2.4.1 Correlation Coefficient	33
Exa 2.5.3 Independent Random Variables	33
Exa 2.6.2 Several Random Variable Problem	34
Exa 3.1.1 Binomial Distribution	35
Exa 3.1.2 Binomial Distribution	35
Exa 3.2.1 Poisson Distribution	36
Exa 3.2.2 Poisson Distribution	36

Exa 3.2.3	Poisson Distribution	36
Exa 3.2.4	Poisson Distribution	37
Exa 3.3.5	Chi Square Distribution	37
Exa 3.4.3	Normal Distribution	37
Exa 3.4.4	Normal Distribution	38
Exa 3.4.5	Finding mu and sigma	38
Exa 3.5.3	Population Normal Distribution Example	38
Exa 4.1.5	Hair Colour Problem	40
Exa 4.1.6	Simulated Poisson Variates	40
Exa 4.1.7	Histogram For Normally Generated Data	41
Exa 4.2.4	Two Sample Confidence Interval Problem	41
Exa 4.2.5	Confidence Intervals	42
Exa 4.4.3	Order Statistics of Random Sample	42
Exa 4.4.4	Five Number Summary	43
Exa 4.4.5	Box And Quantile Plots	43
Exa 4.5.2	Test for Binomial Proportion of Success	44
Exa 4.5.5	Box And Quantile Plots	44
Exa 4.6.4	Randomized Test	45
Exa 4.6.5	P value of Normal Distribution	46
Exa 4.7.1	Goodness Of Fit	46
Exa 4.7.2	Goodness Of Fit	47
Exa 4.8.1	Pi Estimation	48
Exa 4.8.4	Pi Estimation By Monte Carlo Integration	48
Exa 4.8.7	Probability Of Acceptance	49
Exa 4.9.1	Bootstrap	50
Exa 4.9.2	Bootstrap	50
Exa 4.9.3	Bootstrap	51
Exa 4.10.1	Tolerance Interval	52
Exa 5.2.6	Distribution Comparison	53
Exa 5.3.2	Sample Probability	53
Exa 5.3.3	Normal Approximation To Binomial Distribution	54
Exa 5.3.4	Sample Probability	54
Exa 6.2.5	ARE	56
Exa 8.1.1	Multivariate Random Variable Hypotheses	57
Exa 8.1.2	Hypothesis	57
Exa 8.1.3	Simple Hypotheses	58
Exa 8.2.1	PDF Hypothesis	58
Exa 10.2.2	ARE Normal Distribution	60

Exa 10.2.3 ARE Laplace Distribution	60
Exa 10.2.5 Golden Rectangle	61
Exa 10.3.1 Data of Darwin	61
Exa 10.3.2 ARE at Normal Distribution	62
Exa 10.3.4 Zea Mays Problem	62
Exa 10.4.1 Water Wheel Problem	63
Exa 10.4.2 MWW Estimate	64
Exa 10.5.3 General Rank Scores	65
Exa 10.7.2 Telephone Data Scores	67
Exa 10.8.1 Olympic Race Problem	68
Exa 10.8.2 Olympic Race Problem	69
Exa 11.2.6 Bayesian Testing Procedures	70
Exa 11.4.2 Gibbs Sampler	71

Chapter 1

Probability And Distributions

R code Exa 1.1.1 Coin Toss

```
1 #Page no. 1  
2  
3 library(prob)  
4 tosscoin(2)
```

R code Exa 1.1.2 Dice Sample Space

```
1 #Page no 1  
2  
3 sample_space_two_die<-expand.grid(x=1:6, y=1:6)  
4 sample_space_two_die
```

R code Exa 1.1.3 Dice Relative Frequency

```
1 #Page no 2  
2
```

```
3 f<-60
4 N<-400
5 rf<-f/N
6 p<-rf
7 rf
8 p
```

R code Exa 1.2.3 Union

```
1 #Page no 4
2
3 c1<-c(8:11)
4 c2<-c(0:10)
5 c3<-union(c2,c1)
6 c3
```

R code Exa 1.2.4 Union

```
1 #Page no. 4
2
3 c1<-c(0:1)
4 c2<-c(-1:2)
5 c3<-union(c1,c2)
6 setequal(c2,c3)
```

R code Exa 1.2.5 Union

```
1 #Page no. 4
2
3 c1<-sample(1000,size=10)
```

```
4 c2<-c(NULL)
5 c3<-union(c1,c2)
6 setequal(c1,c3)
```

R code Exa 1.2.6 Union

```
1 #Page no. 4
2
3 c<-sample(1000, size=10)
4 c1<-union(c,c)
5 setequal(c1,c)
```

R code Exa 1.2.8 Intersection of Sets

```
1 #Page no. 4
2
3 c1<-c('0,0','0,1','1,1')
4 c2<-c('1,1','1,2','2,1')
5 ans<-intersect(c1,c2)
6 ans
```

R code Exa 1.2.10 Intersection of Sets

```
1 #Page no. 4
2
3 c<-sample(1000, size=10)
4 null<-c(NULL)
5 c1<-intersect(c,c)
6 c2<-intersect(c,null)
7 setequal(c1,c)
8 setequal(null,c2)
```

R code Exa 1.2.12 Venn Diagram

```
1 #Page no. 5
2
3 library(VennDiagram)
4 grid.newpage()
5 draw.pairwise.venn(5,5,2,fill="gray",category = c("C1","C2"))
6 grid.newpage()
7 draw.pairwise.venn(5,5,2,fill=c("red","blue"),
category = c("C1","C2"))
```

R code Exa 1.2.13 Coin Sample Space

```
1 #Page no 5
2
3 library(prob)
4 toss<-tosscoin(4)
5 toss
6 set_c<-c()
7 for (i in c(1:16)) {
8   count<-0
9   if(toss[i,1]== 'H')
10   {
11     count=count+1
12   }
13   if(toss[i,2]== 'H')
14   {
15     count=count+1
16   }
17   if(toss[i,3]== 'H')
```

```
18     {
19         count=count+1
20     }
21     if(toss[i,4]=='H')
22     {
23         count=count+1
24     }
25     set_c<-c(set_c,count)
26 }
27 set_c<-unique(set_c)
28 set_c
```

R code Exa 1.2.15 Set Complement

```
1 #Page no. 6
2
3 c<-c(0,1,2,3,4)
4 c1<-c(0,1)
5 complement<-setdiff(c,c1)
6 complement
```

R code Exa 1.2.16 Set Operations

```
1 #Page no. 6
2
3 c<-c(0,1,2,3,4)
4 c1<-c(0,1)
5 comp<-setdiff(c,c1)
6 comp2<-setdiff(c,comp)
7
8 set<-union(c1,comp)
9 setequal(set,c)
10
```

```
11 set<-intersect(c1,comp)
12 setequal(set,NULL)
13
14 set<-union(c1,c)
15 setequal(set,c)
16
17 set<-intersect(c1,c)
18 setequal(set,c1)
19
20 setequal(comp2,c1)
```

R code Exa 1.2.17 De Morgans Law

```
1 #Page no. 6
2
3 c1<-c(0,1,2)
4 c2<-c(2,3,4,5)
5 c<-union(c1,c2)
6 c1comp<-setdiff(c,c1)
7 c2comp<-setdiff(c,c2)
8
9 set<-intersect(c1,c2)
10 lhs<-setdiff(c,set)
11 rhs<-union(c1comp,c2comp)
12 setequal(lhs,rhs)
13
14 set<-union(c1,c2)
15 lhs<-setdiff(c,set)
16 rhs<-intersect(c1comp,c2comp)
17 setequal(lhs,rhs)
```

R code Exa 1.2.18 Set Functions

```
1 #Page no. 6
2
3 C<-c(0:4)
4 Qc<-0
5 for (i in C)
6 {
7   if(i>0)
8   {
9     Qc<-Qc+1
10  }
11 }
12 Qc
13
14 C<-c(-2,-1)
15 Qc<-0
16 for (i in C)
17 {
18   if(i>0)
19   {
20     Qc<-Qc+1
21   }
22 }
23 Qc
24
25 inf<-9999999
26 C<-c(-inf:5)
27 Qc<-0
28 for (i in C)
29 {
30   if(i>0)
31   {
32     Qc<-Qc+1
33   }
34 }
35 Qc
```

R code Exa 1.2.21 Functions

```
1 #Page no. 7
2
3 f<-function(x)
4 {
5   ((1/2)^x)*(x>0)
6 }
7 C<-c(0:3)
8 Qc<-f(C)
9 sum(Qc)
```

R code Exa 1.2.23 1D Set Integration

```
1 #Page no 7
2
3 library(pracma)
4 f<-function(x)
5 {
6   exp(-x)
7 }
8
9 qc1<-integral(f,0,Inf)
10 qc1
11
12 qc2<-integral(f,1,2)
13 qc2
14
15 qc3<-integral(f,0,3)
16 qc3
17 #The answer may vary due to difference in
   representation.
```

R code Exa 1.3.1 Set Probability

```
1 #Page no. 13
2
3 p<-1/36
4 c1<-c('1,1','2,1','3,1','4,1','5,1')
5 c2<-c('1,2','2,2','3,2')
6 P1<-length(c1)*p
7 P2<-length(c2)*p
8 P3<-length(union(c1,c2))*p
9 P4<-length(intersect(c1,c2))*p
10 P1
11 P2
12 P3
13 P4
14 #The answer may vary due to difference in
   representation
```

R code Exa 1.3.2 Set Probability

```
1 #Page no. 13
2
3 C<-c('H,H','H,T','T,H','T,T')
4 c1<-c('H,H','H,T')
5 c2<-c('H,H','T,H')
6 p1<-length(c1)/length(C)
7 p2<-length(c2)/length(C)
8 p3<-length(intersect(c1,c2))/length(C)
9 p4<-length(union(c1,c2))/length(C)
10 p1
11 p2
```

```
12 p3
13 p4
14 #The answer may vary due to difference in
   representation
```

R code Exa 1.3.4 Poker Question

```
1 #Page no. 15
2
3 k<-52
4 r1<-13
5 r2<-4
6 E1<-r1/k
7 E2<-r2/k
8 E1
9 E2
10 PE1<-choose(4,1)*choose(13,5)/choose(52,5)
11 round(PE1,5)
12 PE2<-choose(13,1)*choose(4,3)*choose(12,2)*choose
   (4,1)^2/choose(52,5)
13 round(PE2,4)
14 PE3<-choose(4,3)*choose(4,2)/choose(52,5)
15 round(PE3,7)
```

R code Exa 1.4.1 Conditional Probabilty

```
1 #Page no. 22
2
3 number_of_cards<-52
4 number_of_spades<-13
5
6 CP2=choose(number_of_spades,5)/choose(number_of_
   cards,5)
```

```
7 CP1=(choose(number_of_spades,4)*choose(39,1)+choose(
    number_of_spades,5))/choose(number_of_cards,5)
8 ans<-round(CP2/CP1,4)
9 ans
```

R code Exa 1.4.2 Conditional Probabilty Chip Problem

```
1 #Page no. 22
2
3 number_of_chips<-8
4 redchips<-3
5 bluechips<-5
6 pc1<-redchips/number_of_chips
7 pc2c1<-bluechips/(number_of_chips-1)
8 ans<-pc1*pc2c1
9 round(ans,4)
```

R code Exa 1.4.3 Conditional Probabilty Card Problem

```
1 #Page no. 22
2
3 number_of_cards<-52
4 number_of_spades<-13
5
6 CP1=choose(number_of_spades,2)*choose(number_of_
    cards-number_of_spades,3)/choose(number_of_cards
    ,5)
7 CP1=round(CP1,4)
8 PC2givenC1=11/47
9
10 ans=round(CP1*PC2givenC1,4)
11 ans
```

R code Exa 1.4.4 Cards In Order

```
1 #Page no. 23
2
3 n<-52
4 spade<-13
5 heart<-13
6 diamond<-13
7 club<-13
8 p<-(spade/n)*(heart/(n-1))*(diamond/(n-2))*(club/(n
-3))
9 round(p,4)
```

R code Exa 1.4.5 Bayes Theorem

```
1 #Page no. 23
2
3 c1_red<-3
4 c1_blue<-7
5 c2_red<-8
6 c2_blue<-2
7 PC1<-2/6
8 PC2<-4/6
9 PCgivenC1<-c1_red/(c1_blue+c1_red)
10 PCgivenC2<-c2_red/(c2_blue+c2_red)
11 PC1givenC<-PC1*PCgivenC1/((PC1*PCgivenC1)+(PC2*
    PCgivenC2))
12 PC2givenC<-1-PC1givenC
13 PC1givenC
14 PC2givenC
15 #The answer may vary due to difference in
   representation
```

R code Exa 1.4.6 Plant Problem

```
1 #Page no. 24
2
3 PC1<-0.1
4 PC2<-0.5
5 PC3<-0.4
6 PCC1<-0.01
7 PCC2<-0.03
8 PCC3<-0.04
9 PC1C<-(PC1*PCC1)/(PC1*PCC1+PC2*PCC2+PC3*PCC3)
10 PC1C
11 #The answer may vary due to difference in
   representation
```

R code Exa 1.4.7 Bayes Theorem Child Abuse Problem

```
1 #Page no. 24
2
3 A<-0.01
4 N<-0.99
5 PNd_A<-0.04
6 PAd_N<-0.05
7 PAd_A<-0.96
8 PNd_N<-0.95
9 PAd<-PAd_A*A+PAd_N*N
10 PAd
11
12 PA_Ad<-PAd_A*A/PAd
13 round(PA_Ad,4)
```

R code Exa 1.4.8 Sum of 2 Die

```
1 #Page no. 26
2
3 n1<-c(rep(1,6),rep(2,6),rep(3,6),rep(4,6),rep(5,6),
      rep(6,6))
4 n2<-c(rep(1:6,6))
5 count<-n1+n2
6 sum<-as.data.frame(table(count))
7 prob<-sum$Freq/length(n1)
8 prob
9 #The answer may vary due to difference in
   representation
```

R code Exa 1.4.9 Mutual Independence

```
1 #Page no 27
2
3 sample<-c(1:4)
4 sample2<-expand.grid(x=1:4, y=1:4)
5 sample
6 sample2
7 c1<-4
8 c2<-1
9 c3<-1
10 pc1<-c1/(length(sample)*length(sample))
11 pc2<-c2/length(sample)
12 pc3<-c3/length(sample)
13 pc1
14 pc2
15 pc3
16 pc1pc2pc3<-1/(length(sample)*length(sample))
```

```
17 PC<-pc1*pc2*pc3
18
19 PC
20 pc1c2c3
21 #The answer may vary due to difference in
   representation.
```

R code Exa 1.4.10 Coin Probability

```
1 #Page no. 27
2
3 H<-1/2
4 T<-1-H
5 P1<-H*H*T*H
6 P2<-T*T*H
7 P3<-1-(T*T*T*T)
8 P1
9 P2
10 P3
11 #The answer may vary due to difference in
   representation
```

R code Exa 1.4.11 Mutually Independent Events

```
1 #Page no. 27
2
3 k1<-0.01
4 k2<-0.03
5 k3<-0.08
6 Pfailure<-k1*k2*k3
7 Pnotfailure<-1-Pfailure
8 Pfailure
9 Pnotfailure
```

R code Exa 1.5.1 Random Variable Pmf

```
1 #Page no. 32
2
3 n1<-c(rep(1,6),rep(2,6),rep(3,6),rep(4,6),rep(5,6),
      rep(6,6))
4 n2<-c(rep(1:6,6))
5 count<-n1+n2
6 sum<-as.data.frame(table(count))
7 prob<-sum$Freq/length(n1)
8 sum$count
9 prob
10 p1<-prob[6]+prob[10]
11 p1
12 p2<-prob[1]+prob[2]+prob[11]
13 p2
14 #The answer may vary due to difference in
   representation
```

R code Exa 1.5.2 Probability Density Function

```
1 #Page no. 33
2
3 f<-function(x) {(x>0 & x<1)*1}
4 fx1<-integrate(f,lower=0,upper=1/8)
5 fx2<-integrate(f,lower=7/8,upper=1)
6 ans<-fx1$value+fx2$value
7 ans
8 #The answer may vary due to difference in
   representation
```

R code Exa 1.5.3 Step Function Plot

```
1 #Page no. 34
2
3 f<-function(x){(x<1)*0+(x>=1 & x<2)*1/6+(x>=2 & x<3)
  *2/6+(x>=3 & x<4)*3/6+(x>=4 & x<5)*4/6+(x>=5 & x
  <6)*5/6+(x==6)*1}
4 plot(f,xlim=c(0,6),ylim=c(0,1))
```

R code Exa 1.5.4 Cumulative Distribution Function

```
1 #Page no. 34
2
3 f<- function(x) {(x<0)*0+(x>=0 & x<1)*x+(x>=1)*1}
4 plot(f,xlim=c(-1,2),ylim=c(0,1))
```

R code Exa 1.5.5 Mechanical Part CDF Problem

```
1 #Page no. 36
2
3 cdf<-expression(1-exp(-x))
4 pdf<-D(cdf,'x')
5 x<-1
6 ans1<-eval(pdf)
7 x<-3
8 ans2<-eval(pdf)
9 ans<-ans1-ans2
10 round(ans,3)
```

R code Exa 1.5.6 Cumulative Distribution Function

```
1 #Page no. 37
2
3 f<-function(x)
4 {
5   x/2*(x>=0 & x<1)+1*(x>=1)
6 }
7 p1<-f(1/2)-f(-1)
8 p1
9 p2<-f(1)-f(0.99999999)
10 p2
11 #The answer may vary due to difference in
   representation
```

R code Exa 1.5.7 Probability Mass Function

```
1 #Page no 38
2
3 library(rSymPy)
4 sympyStart()
5
6 x<-c(1:10)
7 c<-Var('c')
8 sumpx<-sum(x)*c
9 sumpx
10 sympy("solve ([Eq(55*c,1)],c)")
```

R code Exa 1.5.8 Probability Density Function

```
1 #Page no. 38
2
3 f<-function(x)
4 {
5   ((x^3)/4)*(x>0 & x<2)
6 }
7 ans<-integrate(f,lower=1/4,upper=1)
8 round(ans$value,5)
```

R code Exa 1.6.2 Fuse Problem

```
1 #Page no. 41
2
3 fuses<-100
4 defective<-20
5 ans<-choose(fuses-defective,5)/choose(fuses,5)
6 round(ans,5)
```

R code Exa 1.7.1 Circle Probability Problem

```
1 #Page no. 45
2
3 pdf<-function(x) (2*x)
4 px<-integrate(pdf,lower = 1/4,upper=1/2)
5 px$value
6 #The answer may vary due to difference in
  representation
```

R code Exa 1.7.2 Telephone Probability Problem And Graph

```
1 #Page no. 45
2
3 f=function(x) (1/4*exp(-x/4))
4 px=integrate(f,lower=4,upper=Inf)
5
6 plot(f,type='l',ylab='f(x)',ylim=c(0,0.3),xlim=c
      (0,6))
7 cord.x<-c(4,4,6,6)
8 cord.y<-c(0,f(4),f(6),0)
9 polygon(cord.x,cord.y,col='blue')
10 round(px$value,4)
```

R code Exa 1.7.6 Cumulative Distribution Function

```
1 #Page no. 48
2
3 f1=function(x) {(x<0)*(0)+(x>=0 & x<1)*((x+1)/2)+(x
      >=1)*1}
4 p1=f1(1/2)-0
5 p2=f1(0)-0
6 plot(f1,type="l",ylab="f(x)",xlim=c(0,3),ylim=c(0,2)
      )
7 p1
8 p2
9 #The answer may vary due to difference in
   representation
```

R code Exa 1.7.7 Company Loss Problem

```
1 #Page no 49
2
3 f<-function(y)
4 {
```

```
5      1-(10/(10+y))^3
6  }
7  jump=1-f(10)
8  jump
9
10 #The answer may vary due to difference in
    representation.
```

R code Exa 1.8.2 Random Variable

```
1 #Page no. 53
2
3 x<-c(1,2,3,4)
4 px<-c(0.4,0.1,0.3,0.2)
5 ex=sum(x*px)
6 ex
```

R code Exa 1.8.3 Probability Density Function

```
1 #Page no. 53
2
3 f<-function(x){(x>0 & x<1)*(x*4*x^3)}
4 ex<-integrate(f,lower=0,upper=1)
5 ex$value
6 #The answer may vary due to difference in
    representation
```

R code Exa 1.8.4 Expectation Problem

```
1 #Page no. 55
```

```
2
3 f1<-function(x){(x>0 & x<1)*x*2*(1-x)}
4 f2<-function(x){(x>0 & x<1)*x^2*2*(1-x)}
5 ex<-integrate(f1,lower=-Inf,upper=Inf)
6 ex2<-integrate(f2,lower=-Inf,upper=Inf)
7 ex$value
8 ex2$value
9 ans<-6*ex$value+3*ex2$value
10 ans
11 #The answer may vary due to difference in
    representation
```

R code Exa 1.8.5 Expectation Problem

```
1 #Page no. 55
2
3 px<-function(x1) {x1/6}
4 f<-function(x) {x^3*px(x)}
5 ans<-f(1)+f(2)+f(3)
6 ans
7 #The answer may vary due to difference in
    representation
```

R code Exa 1.8.6 Expectation Problem

```
1 #Page no. 55
2
3 f<-function(x)
4 {
5   1/5*(x>0 & x<5)
6 }
7 f1<-function(x)
8 {
```

```

9     f(x)*x
10    }
11  f2<-function(x)
12  {
13    f(x)*(5-x)
14  }
15  f3<-function(x)
16  {
17    f(x)*(5-x)*x
18  }
19  ex<-integrate(f1,lower = 0,upper = 5)
20  ex$value
21  ex<-integrate(f2,lower = 0,upper = 5)
22  ex$value
23  ex<-integrate(f3,lower = 0,upper = 5)
24  ex$value
25 #The answer may vary due to difference in
   representation

```

R code Exa 1.8.7 Expectation Chip Problem

```

1 #Page no. 56
2
3 px<-function(x1) {choose(3,x1)*choose(2,2-x1)/choose
   (5,2)}
4 f<-function(x) {(8-3*x)*px(x)}
5 ans<-f(0)+f(1)+f(2)
6 ans
7 #The answer may vary due to difference in
   representation

```

R code Exa 1.9.1 Mean And Variance

```
1 #Page no. 59
2
3 pdf<-function(x) {((x>-1 & x<1)*(x+1)/2}
4 mean<-function(x){x*pdf(x)}
5 variance<-function(x){((x^2)*pdf(x))}
6 ans1<-integrate(mean,lower=-Inf,upper=Inf)$value
7 ans2<-integrate(variance,lower=-Inf,upper=Inf)$value
     -(ans1)^2
8 ans1
9 ans2
10 #The answer may vary due to difference in
    representation
```

R code Exa 1.10.1 Chebyshev Inequality

```
1 #Page no 69
2
3 f<-function(x)
4 {
5   (1/(2*sqrt(3)))*(x>-sqrt(3) & x<sqrt(3))
6 }
7 k<-3/2
8 i<-integrate(f,lower = -k,upper = k)
9 p<-1-i$value
10 p
11 #The answer may vary due to difference in
    representation
```

Chapter 2

Multivariate Distributions

R code Exa 2.1.2 Multivariate Random Variable

```
1 #Page no. 75
2
3 library(cubature)
4 pdf<-function(x){(x[1]>0 & x[1]<1 & x[2]>0 & x[2]<1)
  *6*x[1]^2*x[2]}
5 f1<-adaptIntegrate(pdf,lowerLimit = c(0,1/3),
  upperLimit = c(3/4,2))
6 round(f1$integral,3)
7 #The answer may vary due to difference in
  representation
```

R code Exa 2.1.3 Chips From Bowl Problem

```
1 #Page no 77
2
3 p11<-1/10
4 p12<-1/10
5 p21<-1/10
```

```
6 p22<-2/10
7 p31<-2/10
8 p32<-3/10
9
10 jointp<-matrix(c(p11,p21,p31,p12,p22,p32),ncol=2)
11 jointp
12
13 px1<-apply(jointp,1,sum)
14 px1
15
16 px2<-apply(jointp,2,sum)
17 px2
```

R code Exa 2.1.4 Joint PDF Problem

```
1 #Page no. 78
2
3 library(pracma)
4 f<-function(x1,x2)
5 {
6   (x1>0 & x1<1 & x2>0 & x2<1)*(x1+x2)
7 }
8 P1<-integral2(f,0,0.5,0,1)
9 P1$Q
10
11 ymax<-function(x1){1-x1}
12 P2<-integral2(f,0,1,0,ymax)
13 P2$Q
14 #The answer may vary due to difference in
   representation
```

R code Exa 2.1.5 Multivariate Expectation Problem

```

1 #Page no. 80
2
3 library(pracma)
4 f<-function(x2,x1){8*x1*x2}
5 ymax<-function(x2){x2}
6 e1<-function(x2,x1){x1*(x2^2)*f(x2,x1)}
7 E1<-integral2(e1,0,1,0,ymax)
8 E1$Q
9
10 e2<-function(x2,x1){x2*f(x2,x1)}
11 E2<-integral2(e2,0,1,0,ymax)
12 E2$Q
13
14 pdf<-function(x2){4*(x2)^3}
15 ex2<-function(x2){x2*pdf(x2)}
16 integral(ex2,0,1)
17
18 E3<-(7*E1$Q)+(5*E2$Q)
19 E3
20 #The answer may vary due to difference in
   representation

```

R code Exa 2.1.6 Multivariate Expectation Problem

```

1 #Page no. 81
2
3 library(pracma)
4 pdf<-function(y){2*y}
5 e1<-function(y){y*pdf(y)}
6 E1<-integrate(e1,lower=0,upper=1)
7 E1$value
8
9 f<-function(x2,x1){8*x1*x2}
10 e2<-function(x2,x1){(x1/x2)*f(x2,x1)}
11 ymax<-function(x2){x2}

```

```
12 E2<-integral2(e2,0,1,0,ymax)
13 E2$Q
14 #The answer may vary due to difference in
   representation
```

R code Exa 2.3.1 Mean Variance And Probability

```
1 #Page no. 96
2
3 library(pracma)
4 f1<-function(x1){2*(1-x1)}
5 f2<-function(x2){1/x2}
6 f3<-function(x1){f2(3/4)}
7 p1<-integral(f3,0,1/2)
8 p1
9
10 p2<-integral(f1,0,1/2)
11 p2
12 #The answer may vary due to difference in
   representation
```

R code Exa 2.3.2 Mean And Variance

```
1 #Page no. 97
2
3 g<-function(y){81/8*y^2}
4 ey<-function(y){y*g(y)}
5 Ey<-integral(ey,0,2/3)
6 Ey
7 ey2<-function(y){y^2*g(y)}
8 Ey2<-integral(ey2,0,2/3)
9 var<-Ey2-(Ey)^2
10 var
```

```
11 #The answer may vary due to difference in  
representation
```

R code Exa 2.4.1 Correlation Coefficient

```
1 #Page no. 102  
2  
3 library(pracma)  
4 f<-function(x,y){x+y}  
5 ex<-function(x,y){x*f(x,y)}  
6 Ex1<-integral2(ex,0,1,0,1)  
7 Ex1$Q  
8 ex2<-function(x,y){x^2*f(x,y)}  
9 Ex2<-integral2(ex2,0,1,0,1)  
10 Ex2$Q  
11 varx<-Ex2$Q-(Ex1$Q^2)  
12 varx  
13 Ey1<-Ex1  
14 vary<-varx  
15 exy<-function(x,y){x*y*f(x,y)}  
16 Exy<-integral2(exy,0,1,0,1)  
17 num<-Exy$Q-(Ex1$Q*Ex1$Q)  
18 num  
19 den<-sqrt(vary*varx)  
20 corr<-num/den  
21 corr  
22 #The answer may vary due to difference in  
representation
```

R code Exa 2.5.3 Independent Random Variables

```
1 #Page no. 113  
2
```

```

3 library(pracma)
4 f1<-function(x1,x2){x1+x2}
5 f2<-function(x1){x1+1/2}
6 f3<-function(x2){1/2+x2}
7 p1<-integral2(f1,0,1/2,0,1/2)$Q
8 p2<-integral(f2,0,1/2)
9 p3<-integral(f2,0,1/2)
10 p1
11 p2
12 p3
13 p2*p3
14 #The answer may vary due to difference in
   representation

```

R code Exa 2.6.2 Several Random Variable Problem

```

1 #Page no. 121
2
3 library(pracma)
4 pdf<-function(x){2*x}
5 f<-function(x1,x2,x3){pdf(x1)*pdf(x2)*pdf(x3)}
6 f1<-function(x1,x2,x3){(5*x1*(x2^3)+3*x2*(x3)^4)*f(
  x1,x2,x3)}
7 E<-integral3(f1,0,1,0,1,0,1)
8 E
9
10 p<-integral3(f,0,1/2,0,1/2,0,1/2)
11 p
12 #The answer may vary due to difference in
   representation

```

Chapter 3

Some Special Distributions

R code Exa 3.1.1 Binomial Distribution

```
1 #Page no. 141
2
3 n<-7
4 p1<-pbinom(1,n,1/2,lower.tail = T)
5 p2<-dbinom(5,n,1/2)
6 p1
7 p2
8 #The answer may vary due to difference in
   representation
```

R code Exa 3.1.2 Binomial Distribution

```
1 #Page no. 141
2
3 n<-5
4 p<-1/3
5 q<-1-p
6 mean<-n*p
```

```
7 var<-n*p*q
8 mean
9 var
10 #The answer may vary due to difference in
    representation
```

R code Exa 3.2.1 Poisson Distribution

```
1 #Page no. 152
2
3 lambda<-2
4 p<-1-dpois(0,2)
5 round(p,3)
```

R code Exa 3.2.2 Poisson Distribution

```
1 #Page no. 153
2
3 lambda<-4
4 p<-dpois(3,lambda)
5 round(p,3)
```

R code Exa 3.2.3 Poisson Distribution

```
1 #Page no. 153
2
3 lambda<-1/1000
4 mean<-3000*lambda
5 p<-1-ppois(4,mean)
6 round(p,3)
```

R code Exa 3.2.4 Poisson Distribution

```
1 #Page no. 154
2
3 lambda<-1/1000
4 mean<-3000*lambda
5 newmean<-3*mean
6 p<-1-ppois(14,newmean)
7 round(p,3)
```

R code Exa 3.3.5 Chi Square Distribution

```
1 #Page no. 160
2
3 p1<-round(pchisq(20.5,df=10),3)
4 p2<-round(pchisq(3.5,df=10),3)
5 p<-p1-p2
6 round(p,2)
7 #The answer may slightly vary due to rounding off
   values
```

R code Exa 3.4.3 Normal Distribution

```
1 #Page no. 172
2
3 mean=2
4 v=25
5 sd=sqrt(v)
6 p1<-pnorm(10,mean,sd)-pnorm(0,mean,sd)
```

```
7 p1
8
9 p2<-pnorm(1,mean,sd)-pnorm(-8,mean,sd)
10 round(p2,3)
```

R code Exa 3.4.4 Normal Distribution

```
1 #Page no 172
2
3 p<-pnorm(2)-pnorm(-2)
4 round(p,3)
```

R code Exa 3.4.5 Finding mu and sigma

```
1 #Page no 173
2
3 library(rSymPy)
4 sympyStart()
5
6 m<-Var("m")
7 s<-Var("s")
8 sympy("solve ([Eq(-1.28*s+m,60),Eq(1.64*s+m,90)],(s,m))")
9
10 #The answer may slightly vary due to rounding off
values.
```

R code Exa 3.5.3 Population Normal Distribution Example

```
1 #Page no. 184
```

```
2  
3 p<-pnorm(2)-pnorm(-2)  
4 round(p,3)*100
```

Chapter 4

Some Elementary Statistical Inferences

R code Exa 4.1.5 Hair Colour Problem

```
1 #Page no. 208
2
3 head<-c("Medium","Fair","Dark","Red","Black")
4 count<-c(9418,5789,5678,1319,157)
5 p<-c(0.421,0.259,0.254,0.059,0.007)
6 data<-data.frame(head,count,p)
7 barplot(data$p,names.arg = data$head)
```

R code Exa 4.1.6 Simulated Poisson Variates

```
1 #Page no 209
2
3 variates<-c(2,1,1,1,1,5,1,1,3,0,2,1,1,3,4,
4           2,1,2,2,6,5,2,3,2,4,1,3,1,3,0)
5 j<-c(0,1,2,3,4,5,6)
6 total<-length(variates)
```

```
7 count<-c(2,11,7,5,2,2,1)
8 p<-round(count/total,3)
9 j
10 p
11 barplot(count,names.arg = j)
```

R code Exa 4.1.7 Histogram For Normally Generated Data

```
1 #Page no. 211
2
3 x<-c('10-20','20-30','30-40','40-50','50-60','60-70',
      )
4 count<-c(1,0,3,8,5,3)
5 data<-data.frame(x,count)
6 barplot(count,names.arg=x)
```

R code Exa 4.2.4 Two Sample Confidence Interval Problem

```
1 #Page no 218
2
3 nx<-10
4 ny<-7
5 meanx<-4.2
6 meany<-3.4
7 sx<-49
8 sy<-32
9
10 diff<-meanx-meany
11 sp<-sqrt(((nx-1)*sx)+((ny-1)*sy))/(nx+ny-2)
12 me<-sqrt((1/nx)+(1/ny))*sp*qt(0.95,df=nx+ny-2)
13 interval_lower<-diff-me
14 interval_upper<-diff+me
15 interval_lower
```

16 interval_upper

R code Exa 4.2.5 Confidence Intervals

```
1 #Page no 219
2
3 n1<-100
4 n2<-400
5 y1<-30
6 y2<-80
7 p1<-y1/n1
8 p2<-y2/n2
9 se<-sqrt((p1*(1-p1)/n1)+(p2*(1-p2)/n2))
10 alpha<-(100-95.4)/100
11 ov<-p1-p2
12 m<-qnorm(alpha/2)*se
13 me<-round(m,1)
14 lowerinterval<-round(ov+me,1)
15 upperinterval<-ov-me
16 lowerinterval
17 upperinterval
```

R code Exa 4.4.3 Order Statistics of Random Sample

```
1 #Page no 230
2
3 y1<-expression(z2-z1)
4 y3<-expression(z2)
5 D(y1, 'z1')
6 D(y1, 'z2')
7 D(y3, 'z1')
8 D(y3, 'z2')
9
```

```

10 J<-matrix(c(-1,1,0,1), ncol=2)
11 d<-det(J)
12 d
13
14 library(rSymPy)
15 sympyStart()
16
17 z1<-Var('z1')
18 h<-abs(d)*6*z1
19 h
20
21 sympy("z1=Symbol('z1')")
22 sympy("z2=Symbol('z2')")
23 sympy("integrate(6*z1, (z2, z1, 1))")

```

R code Exa 4.4.4 Five Number Summary

```

1 #Page no 232
2
3 x<-c
  (56,70,89,94,96,101,102,102,102,105,106,108,110,113,116)

4 fivenum(x)
5 middle_data<-median(x)
6 middle_data

```

R code Exa 4.4.5 Box And Quantile Plots

```

1 #Page no . 232
2
3 library(L1pack)
4

```

```

5 x <- c
(56,70,89,94,96,101,102,102,102,105,106,108,110,113,116)

6 q1 <- x[floor(length(x)/4)+1]
7 q2 <- x[floor(length(x)/2)+1]
8 q3 <- x[floor(length(x)/4)+1+floor(length(x)/2)+1]
9 h <- 1.5*(q3-q1)
10 h
11 lf <- q1-h
12 lf
13 uf <- q3+h
14 uf
15 n <- length(x)
16 p <- (1:n)/(n + 1)
17 normalqs <- qnorm(p)
18 y <- sort(x)
19 par(mfrow = c(2, 2))
20 boxplot(y, ylab = "x")
21 plot(normalqs, y)
22 plot(qlaplace(p), y)
23 plot(qexp(p), y)

```

R code Exa 4.5.2 Test for Binomial Proportion of Success

```

1 #Page no 242
2
3 pbinom(11,20,0.7)
4 pbinom(12,20,0.7)

```

R code Exa 4.5.5 Box And Quantile Plots

```

1 #Page no 245
2

```

```

3
4 cross<-c
  (23.5,12,21,22,19.125,21.5,22.125,20.375,18.250,21.625,23.250,21,
5 self<-c
  (17.375,20.375,20,20,18.375,18.625,18.625,15.25,16.5,18,16.25,18,
6 data<-cross-self
7 n <- length(data)
8 xbar<-round(mean(data),2)
9 xbar
10 std<-round(sqrt(var(data)),2)
11 std
12 t_crit<-qt(0.05,n-1,lower.tail = FALSE)
13 t_crit
14 t_stat<-round(xbar/(std/sqrt(n)),2)
15 t_stat
16
17 if(t_crit<t_stat)
18 {
19   cat("Rejected H0")
20 }
21
22 p <- (1:n)/(n + 1)
23 normalqs <- qnorm(p)
24 y <- sort(data)
25 boxplot(y, ylab = "x")
26 plot(normalqs, y)

```

R code Exa 4.6.4 Randomized Test

```

1 #Page no 251
2
3 p1<-round(ppois(2,1,lower.tail = F),3)
4 p2<-round(ppois(3,1,lower.tail = F),3)

```

```
5 p1
6 p2
7
8 alpha<-0.05
9 p3<-(alpha-p2)/(p1-p2)
10 p3
11
12 p4<-p2+dpois(3,1)*p3
13 round(p4,2)
14
15 #The answer may vary due to difference in
   representation.
```

R code Exa 4.6.5 P value of Normal Distribution

```
1 #Page no 252
2
3 n<-25
4 sd2<-4
5 mu<-77
6 mean<-76.1
7 var<-sd2/n
8 z<-(mean-mu)/sqrt(var)
9 a<-0.05
10 pnorm(z)
11 a
```

R code Exa 4.7.1 Goodness Of Fit

```
1 #Page no. 256
2
3 x<-c(1,2,3,4,5,6)
4 p<-1/6
```

```
5 n<-60
6 a<-0.05
7 exp<-c(n*p,n*p,n*p,n*p,n*p,n*p)
8 obs<-c(13,19,11,8,5,4)
9 df<-length(x)-1
10 chisq<-sum((obs-exp)^2/exp)
11 chisq
12 tv<-qchisq(1-a,df)
13 tv
14 if (chisq>tv){
15   cat("Rejected")
16 } else{
17   ("Accepted")
18 }
```

R code Exa 4.7.2 Goodness Of Fit

```
1 #Page no 256
2
3 df<-4
4 a<-0.025
5 n<-80
6 exp<-c(5,15,25,35)
7 obs<-c(6,18,20,36)
8 chisq<-round(sum((obs-exp)^2/exp),2)
9 chisq
10 tv<-round(qchisq(1-a,df-1),2)
11 tv
12 if (chisq>tv){
13   cat("Rejected")
14 } else{
15   ("Accepted")
16 }
```

R code Exa 4.8.1 Pi Estimation

```
1 #Page no 262
2
3 findpi<-function(n)
4 {
5   u1<-runif(n)
6   u2<-runif(n)
7   count<-rep(0,n)
8   check<-u1^2+u2^2-1
9   count [check<0]<-1
10
11 pi<-4*mean(count)
12 se<-1.96*4*sqrt(mean(count)*(1-mean(count))/n)
13 list(pi_estimate=pi,standard_error=se)
14 }
15
16 findpi(100)
17 findpi(500)
18 findpi(1000)
19 findpi(10000)
20 findpi(100000)
21
22 #The answer may slightly vary due to rounding off
values.
```

R code Exa 4.8.4 Pi Estimation By Monte Carlo Integration

```
1 #Page no 265
2
3 findpi<-function(n){
4   samp<-4*sqrt(1-runif(n)^2)
```

```
5 pi<-mean(samp)
6 se<-1.96*sqrt(var(samp)/n)
7 list(pi_estimate=pi,lowerCI=pi-se,upperCI=pi+se)
8 }
9
10 findpi(100)
11 findpi(1000)
12 findpi(10000)
13 findpi(100000)
14
15 #The answer may slightly vary due to rounding off
   values.
```

R code Exa 4.8.7 Probability Of Acceptance

```
1 #Page no 269
2
3 f1<-function(x)
4 {
5   (1+x^2)*exp(-x^2/2)
6 }
7 f2<-function(x)
8 {
9   (pi/sqrt(2*pi))*2*exp(-x^2/2)
10 }
11 M1<-f1(1)
12 M1
13 M2<-f2(1)
14 M2
15 1/M2
16 #The answer may slightly vary due to rounding off
   values.
```

R code Exa 4.9.1 Bootstrap

```
1 #Page no 275
2
3 boot<-function(x,b,alpha){
4   theta<-mean(x)
5   tstar<-rep(0,b)
6   n<-length(x)
7   for(i in 1:b){xstar<-sample(x,n,replace=T)
8     tstar[i]<-mean(xstar)
9   }
10  tstar<-sort(tstar)
11  pick<-round((alpha/2)*(b+1))
12  lower<-tstar[pick]
13  upper<-tstar[b-pick+1]
14  list(mean=theta,lower_conf=lower,upper_conf=upper)
15 }
16 x<-c
17 (131.7,182.7,73.3,10.7,150.4,42.3,22.2,17.9,264.0,154.4,4.3,265.6
18 boot(x,3000,0.1)
19 x1<-c
20 (4.3,4.3,4.3,10.8,10.8,10.8,10.8,17.9,22.5,42.3,48.8,48.8,85.9,13
21 #The answer may slightly vary due to rounding off
values.
```

R code Exa 4.9.2 Bootstrap

```
1 #Page no 278
2
3 x<-c
4 (94.2,111.3,90.0,99.7,116.8,92.2,166.0,95.7,109.3,106.0,111.7,111
```

```

4 y<-c
  (125.5,107.1,67.9,98.2,128.6,123.5,116.5,143.2,120.3,118.6,105.0,
5 boxplot(y,x,border = "black",horizontal = TRUE,names
  =c("Sample 2","Sample 1"))
6 test_statistic<-mean(y)-mean(x)
7 round(test_statistic,2)

```

R code Exa 4.9.3 Bootstrap

```

1
2 #Page no 280
3
4 bootstrap<-function(x,t,b){
5   n<-length(x)
6   v<-mean(x)
7   z<-x-mean(x)+t
8   counter<-0
9   final<-rep(0,b)
10  for(i in 1:b){xstar<-sample(z,n,replace=T)
11    vstar<-mean(xstar)
12    if(vstar >= v){counter<-counter+1}
13    final[i]<-vstar}
14    pvalue<-counter/b
15    list(origtest=v,pvalue=pvalue,final=final)
16  }
17
18 x<-c
  (119.7,104.1,92.8,85.4,108.6,93.4,67.1,88.4,101,97.2,95.4,77.2,10
19 ans<-bootstrap(x,90,3000)
20 hist(ans$final)

```

R code Exa 4.10.1 Tolerance Interval

```
1 #Page no 287
2
3 f<-function(v)
4 {
5   (30*(v^4))*(1-v)
6 }
7 i<-integrate(f, lower=0, upper=0.8)
8 ans<-1-i$value
9 round(ans,2)
```

Chapter 5

Consistency and Limiting Distributions

R code Exa 5.2.6 Distribution Comparison

```
1 #Page no . 303
2
3 n<-50
4 p<-1/25
5 round(pbinom(1,n,p),3)
6 mhu<-n*p
7 poiss<-round(ppois(1,mhu),3)
8 poiss
```

R code Exa 5.3.2 Sample Probability

```
1 #Page no 309
2
3 n<-75
4 mhu<-1/2
5 var<-1/12
```

```
6  sd<-sqrt(var)
7  l1<-sqrt(n)*(0.45-mhu)/sd
8  l2<-sqrt(n)*(0.55-mhu)/sd
9  ans<-round(pnorm(l2)-pnorm(l1),3)
10 ans
```

R code Exa 5.3.3 Normal Approximation To Binomial Distribution

```
1 #Page no. 310
2
3 n<-10
4 p<-1/2
5 mean<-n*p
6 i<-c(0:10)
7 binomial<-dbinom(i,n,p)
8 normal<-dnorm(i,5,2.5)
9 barplot(binomial,names.arg=i)
10 par(new="T")
11 plot(normal,type='l',xlab="",ylab="",xaxt="n",yaxt="n")
```

R code Exa 5.3.4 Sample Probability

```
1 #Page no. 310
2
3 y<-c(48:52)
4 n<-100
5 p<-1/2
6 q<-1-p
7 mean<-n*p
8 var<-n*p*q
9 sd<-sqrt(var)
10 l1<-(47.5-mean)/sd
```

```
11 l2<-(52.5-mean)/sd
12 ans<-round(pnorm(l2)-pnorm(l1),3)
13 ans
14
15 #The answer may slightly vary due to rounding off
   values
```

Chapter 6

Maximum Likelihood Methods

R code Exa 6.2.5 ARE

```
1 #Page no 336
2
3 f1<-function(z)
4 {
5   z^2*exp(-z)
6 }
7 i<-integrate(f1,lower=0,upper=Inf)
8 i$value
```

Chapter 8

Optimal Tests of Hypotheses

R code Exa 8.1.1 Multivariate Random Variable Hypotheses

```
1 #Page no. 430
2
3 n<-5
4 i<-c(0:5)
5 r1<-c(dbinom(i,n,1/2))
6 r2<-c(dbinom(i,n,3/4))
7 r3<-r1/r2
8 df<-data.frame(r1,r2,r3)
9 df
10 P<-df$r2[5]+df$r2[6]
11 P
12 #The answer may vary due to difference in
   representation
```

R code Exa 8.1.2 Hypothesis

```
1 #Page no 435
2
```

```
3 f1<-function(w)
4 {
5   (1/sqrt(2*pi))*exp(-w^2/2)
6 }
7 i<-integrate(f1,lower=-3.355,upper=Inf)
8 i$value
```

R code Exa 8.1.3 Simple Hypotheses

```
1 #Page no. 436
2
3 f1<-function(x)
4 {
5   exp(-1)/factorial(x)
6 }
7 f2<-function(x)
8 {
9   (1/2)^(x+1)
10}
11 p1<-1-(f1(1)+f1(2))
12 p2<-1-(f2(1)+f2(2))
13 round(p1,3)
14 round(p2,3)
```

R code Exa 8.2.1 PDF Hypothesis

```
1 #Page no 439
2
3 f<-function(t)
4 {
5   ((t+9.5)/t)*exp(-9.5/t)
6 }
7 f(2)
```

8 $f(4)$

9 $f(9.5)$

10 #The answer may slightly vary due to rounding off
values.

Chapter 10

Nonparametric and Robust Statistics

R code Exa 10.2.2 ARE Normal Distribution

```
1 #Page no 536
2
3 library(rSymPy)
4 sympyStart()
5
6 phi<-Var('p')
7 num<-phi*phi
8 num
9 dem<-(pi/2)*phi*phi
10 dem
11 expn=num/dem
12 expn
13 #The answer may slightly vary due to rounding off
   values.
```

R code Exa 10.2.3 ARE Laplace Distribution

```
1 #Page no 537
2
3 library(rSymPy)
4 sympyStart()
5
6 b<-Var('b')
7 num<-2*b*b
8 num
9 dem<-b*b
10 dem
11 expn=num/dem
12 expn
```

R code Exa 10.2.5 Golden Rectangle

```
1 #Page no 540
2
3 library(BSDA)
4 data<-c
   (0.553,0.570,0.576,0.601,0.606,0.606,0.609,0.611,0.615,0.628,0.654,
5 md<-median(data)
6 md
7 SIGN.test(data,md=md)
```

R code Exa 10.3.1 Data of Darwin

```
1 #Page no. 544
2
3 p1<-1-psignrank(95,15)
4 round(p1,3)
5
6 mean<-60
```

```
7 den<-sqrt(15*16*31/24)
8 num<-95.5-mean
9 p2<-round(1-pnorm(num/den),3)
10 p2
```

R code Exa 10.3.2 ARE at Normal Distribution

```
1 #Page no. 548
2
3 f<-function(x)
4 {
5   ((1/sqrt(2*pi))*exp(-(x^2)/2))^2
6 }
7 tw_1<-integrate(f, lower=-Inf, upper=Inf)
8 tw_1<-sqrt(12)*tw_1$value
9 tw2<-(1/tw_1)^2
10 sigma<-1
11 are<-sigma^2/tw2
12 round(are,3)
```

R code Exa 10.3.4 Zea Mays Problem

```
1 #Page no 550
2
3 library(Rfit)
4 library(DescTools)
5 data<-c
6   (6.125,-8.375,1.000,2.000,0.750,2.925,3.500,5.125,1.750,3.625,7.000)
7 ans<-median(walsh(data))
8 round(ans,2)
9 conf_level=0.95
```

```

10 alpha=1-conf_level
11 crit_z<-qnorm(1-alpha/2)
12 n<-length(data)
13 c<-(n*(n+1)/4)-(crit_z*sqrt(n*(n+1)*(2*n+1)/24))-1/2
14 c
15
16 conf<-sort(walsh(data))
17 lower_conf<-conf[round(c)+1]
18 lower_conf
19 upper_conf<-conf[length(conf)-round(c)+1]
20 upper_conf

```

R code Exa 10.4.1 Water Wheel Problem

```

1 #Page no 553
2
3 x<-c(2.3,0.3,5.2,3.1,1.1,0.9,2.0,0.7,1.4,0.3)
4 y<-c(0.8,2.8,4.0,2.4,1.2,0.0,6.2,1.5,28.8,0.7)
5 combine<-sort(c(x,y))
6 ranks<-rank(combine)
7 ranky<-c()
8 for (i in y){
9   j<-match(i,combine)
10  ranky<-c(ranky,ranks[j])
11 }
12 w<-sum(ranky)
13 w
14 m<-105
15 v<-175
16 z<-(w-m)/sqrt(v)
17 z
18 p<-2*(1-pnorm(z))
19 p
20 print("Ho accepted")

```

R code Exa 10.4.2 MWW Estimate

```
1 #Page no 558
2
3 x<-c(2.3,0.3,5.2,3.1,1.1,0.9,2.0,0.7,1.4,0.3)
4 y<-c(0.8,2.8,4.0,2.4,1.2,0.0,6.2,1.5,28.8,0.7)
5 combine<-sort(c(x,y))
6 ranks<-rank(combine)
7 ranky<-c()
8 for (i in y){
9   j<-match(i,combine)
10  ranky<-c(ranky,ranks[j])
11 }
12 w<-sum(ranky)
13 m<-105
14 v<-175
15 z<-(w-m)/sqrt(v)
16 mww<-1/z
17 mww
18
19 conf_level=0.95
20 alpha=1-conf_level
21 crit_z<-qnorm(1-alpha/2)
22 n1<-length(x)
23 n2<-length(y)
24 c<-(n1*n2/2)-(crit_z*sqrt(n1*n2*((n1+n2)-1)/12))-1/2
25 c
26
27 dif<-c()
28 for(i in y){
29   dif<-c(dif,i-x)
30 }
31 dif<-sort(dif)
32
```

```
33 lower_conf<-dif[round(c)+1]
34 lower_conf
35 upper_conf<-dif[length(dif)-round(c)+1]
36 upper_conf
```

R code Exa 10.5.3 General Rank Scores

```
1 #Page no 566
2
3 library(rcompanion)
4 library(exactRankTests)
5 library(ggplot2)
6
7 x<-c
8   (51.9,56.9,45.2,52.3,59.5,41.4,46.4,45.1,53.9,42.9,41.5,55.2,32.9
9
10 y<-c
11   (59.2,49.1,54.4,47,55.9,34.9,62.2,41.6,59.3,32.7,72.1,43.8,56.8,7
12
13 ggplot(data.frame(x=x), aes(x = x)) + geom_dotplot(
14   binwidth = 0.15, dotsize = 1.5) + theme_bw() +
15   labs(title = "Sample 1", x = "", y = "")
16 ggplot(data.frame(y=y), aes(x = y)) + geom_dotplot(
17   binwidth = 0.15, dotsize = 2.5) + theme_bw() +
18   labs(title = "Sample 2", x = "", y = "")
19
20 w<-wilcox.test(y,x,mu=0,alt="two.sided",conf.int=T,
21   conf.level=0.95,paired=F,exact=T,correct=T)
22 n_score<-cscores(c(y,x),type="NormalQuantile")
23 X<-sum(n_score[seq(along=x)])
24 normal_p_value<-pperm(X, n_score, length(x),
25   alternative="two.sided",simulate = TRUE)
```

```

19 results1 <- data.frame(mean(y) - mean(x),
20                           round(unname(t.test(y,x)$
21                                         statistic),2),
22                           round(t.test(y,x)$p.value,2),
23                           mean(y) - mean(x))
24 colnames(results1) <- c("Test Statistic",
25                           "Standardized","p-Value","Estimate of ")
26 rownames(results1) <- "Student t"
27 results2 <- data.frame(sum(rank(c(x,y))[16:30]),
28                           round(qnorm(w$p.value/2)*
29                                         -1,2),
30                           round(w$p.value,3),
31                           w$estimate)
32 colnames(results2) <- c("Test Statistic",
33                           "Standardized","p-Value","Estimate of ")
34 rownames(results2) <- "Wilcoxon"
35 results3 <- data.frame(round(X,2),
36                           qnorm(normal_p_value/2)*-1,
37                           normal_p_value,
38                           median(outer(y,x,'-')))
39 colnames(results3) <- c("Test Statistic",
40                           "Standardized","p-Value","Estimate of ")
41 rownames(results3) <- "Normal scores"
42 results1
43 results2
44 results3
45
46 x2<-x
47 x2[5]<-95.5
48 round(unname(t.test(y,x2)$statistic),2)
49 round(t.test(y,x2)$p.value,2)
50
51 w2<-wilcox.test(y,x2,mu=0,alt="two.sided",conf.int=T
52                   ,conf.level=0.95,paired=F,exact=T,correct=T)
53 w2$p.value<-round(w2$p.value,2)
54 round(qnorm(w2$p.value/2)*-1,2)
55 w2$p.value

```

```

51 n_score2<-cscores(c(y,x2),type="NormalQuantile")
52 X2<-sum(n_score2[seq(along=x2)])
53 normal_p_value2<-pperm(X2, n_score2, length(x2),
  alternative="two.sided",simulate = TRUE)
54 normal_p_value2
55 round(qnorm(normal_p_value2/2)*-1,2)
56
57 #The answer may slightly vary due to rounding off
  values

```

R code Exa 10.7.2 Telephone Data Scores

```

1 #Page no. 578
2
3 year<-c(50:73)
4 call<-c
  (0.44,0.47,0.47,0.59,0.66,0.73,0.81,0.88,1.06,1.2,1.35,1.49,
5
  1.61,2.12,11.9,12.4,14.2,15.9,18.2,21.2,4.3,2.4,2.7,2.9)
6 plot(year,call,xlab="Year",ylab="Number of Calls")
7 ls<-lsfit(year,call)
8 coeff<-ls$coefficients
9 coeff
10
11 y<-function(x)
12 {
13   -26+0.504*x
14 }
15 y1<-y(year)
16 y1
17 par(new=T)
18 lines(y=y1,x=year,type='l',xlab="",ylab="",xaxt="n",
  yaxt="n")
19 wilcox.test(year,call)

```

```

20 y<-function(x)
21 {
22   -7.15+0.145*x
23 }
24 y2<-y(year)
25 y2
26 par(new=T)
27 lines(y=y2,x=year,type='l',xlab=""',ylab=""',xaxt="n",
  yaxt="n",lty=2)

```

R code Exa 10.8.1 Olympic Race Problem

```

1 #Page no 584
2
3 library(asympTest)
4 x<-c
  (373.2,246,245.4,252,243.4,236.8,241.8,233.6,233.2,231.2,227.8,229
5      221.2,215.6,218.1,214.9,216.3,219.2,218.4)
6 y<-c
  (3530,3585,5333,3084,3318,2215,1956,2483,1977,1896,1759,2092,1383
7      731,1226,740,595,663)
8 n<-length(x)+length(y)
9 ans<-cor.test(x,y,method=c("kendall"))
10 k<-ans$estimate
11 k
12 asym<-asymp.test(x,y)
13 round(asym$p.value,4)
14 varhk<-2*((2*n)+5)/(9*n*(n-1))
15 zk<-k/sqrt(varhk)
16 zk
17
18 #The answer may slightly vary due to rounding off
  values

```

R code Exa 10.8.2 Olympic Race Problem

```
1 #Page no 586
2
3 library(asympTest)
4 x<-c
  (373.3 ,246 ,245.4 ,252 ,243.4 ,236.8 ,241.8 ,233.6 ,233.2 ,231.2 ,227.8 ,229
5      221.2 ,215.6 ,218.1 ,214.9 ,216.3 ,219.2 ,218.4)
6 y<-c
  (3530 ,3585 ,5333 ,3084 ,3318 ,2215 ,1956 ,2483 ,1977 ,1896 ,1759 ,2092 ,1383
7      731 ,1226 ,740 ,595 ,663)
8 ans<-cor.test(x,y,method=c("spearman"),alternative =
  "two.sided")
9 r<-ans$estimate
10 r
11 n<-length(x)-1
12 asym<-r*sqrt(n)
13 asym
14 ans$p.value
15 print("Rejected Ho")
16
17 #The answer may slightly vary due to rounding off
  values
```

Chapter 11

Bayesian Statistics

R code Exa 11.2.6 Bayesian Testing Procedures

```
1 #Page no 616
2
3
4 alpha<-10
5 beta<-1.2
6 avg<-alpha*beta
7 avg
8 variance<-alpha*beta^2
9 variance
10 std<-sqrt(variance)
11 range<-seq(0,avg+5*std,0.01)
12 prior<-dgamma(range,alpha,rate=1/beta)
13 plot(range,prior,type='l',xlim=c(0,25))
14
15 data<-c
  (11,7,11,6,5,9,14,10,9,5,8,10,8,10,12,9,3,12,14,4)

16 mean(data)
17 suff_stat<-sum(data)
18 suff_stat
19 n<-length(data)
```

```

20 n
21
22 new_alpha<-suff_stat+alpha
23 new_beta<-beta/(n*beta+1)
24 new_alpha
25 new_beta
26 new_avg<-new_alpha*new_beta
27 new_avg
28 new_variance<-new_alpha*new_beta^2
29 new_variance
30 new_std<-sqrt(new_variance)
31 range<-seq(0,avg+5*std,0.01)
32 posterior<-dgamma(range,new_alpha,rate=1/new_beta)
33 plot(range,posterior,type='l',xlim=c(0,25))
34
35 p1<-round(pgamma(10,new_alpha,1/new_beta),4)
36 p2<-1-p1
37 p1
38 p2
39 if(p1>p2)
40 {
41   print("Accept H0")
42 }else
43 {
44   print("Reject H0")
45 }
46 conf_int<-round(qgamma(c(0.025, 0.975),shape=new_
    alpha,rate=1/new_beta),2)
47 conf_int
48 #The answer may slightly vary due to rounding off
  values.

```

R code Exa 11.4.2 Gibbs Sampler

1 #Page no 629

```

2
3 gibbsr2 = function(alpha,m,n){
4   x0 = 1
5   yc = rep(0,m+n)
6   xc = c(x0,rep(0,m-1+n))
7   for(i in 2:(m+n)){yc[i] = rgamma(1,alpha+xc[i-1],2)
8   xc[i] = rpois(1,yc[i])}
9   y1=yc[1:m]
10  y2=yc[(m+1):(m+n)]
11  x1=xc[1:m]
12  x2=xc[(m+1):(m+n)]
13  list(y1=y1,y2=y2,x1=x1,x2=x2)
14 }
15
16 g<-gibbsr2(10,3000,6000)
17 mean(g$x1)
18 mean(g$y1)
19 var(g$x1)
20 var(g$y1)

```
